# Deep Learning - Part 1

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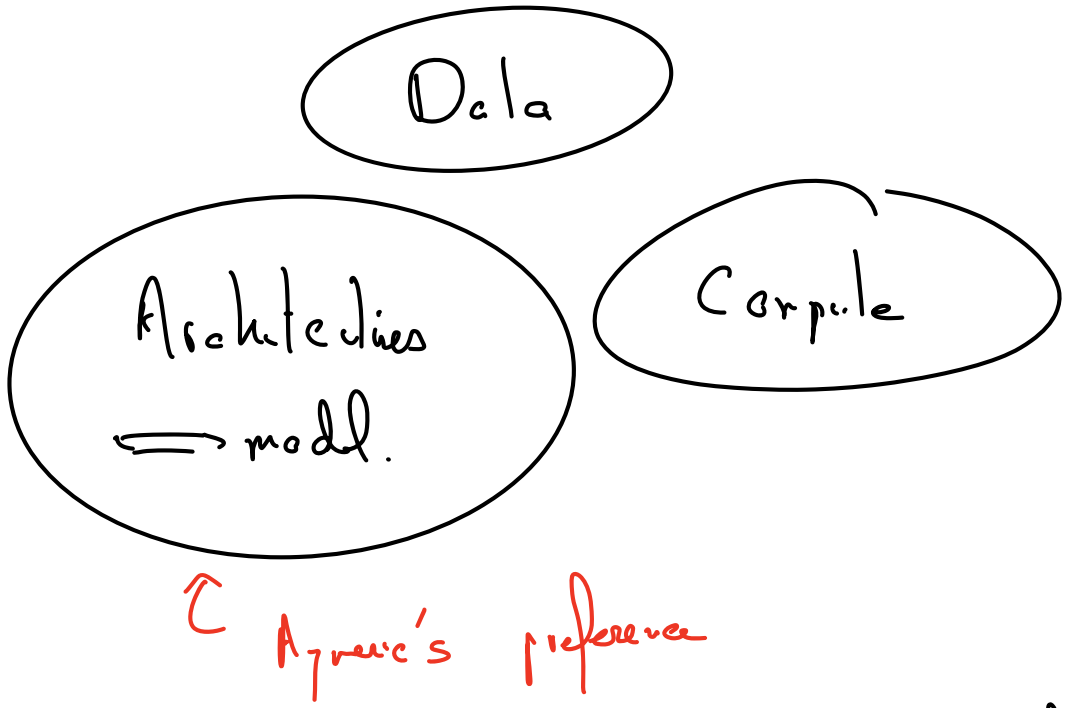
DS4M - April 2025

1. Goals




2. The first Neural Network: the Perceptron
   - Logistic regression
   - Discovering Fully connected NN through the NNPG.
   - Formalization, advanced topics
   - Further discussion

1/3 people that drove the progress

3. Other Architectures — CNN convol. NN; RNN, Transformers too

w.
- Compute
- Data

4. Lab Pytorch and discuss°

# Outline

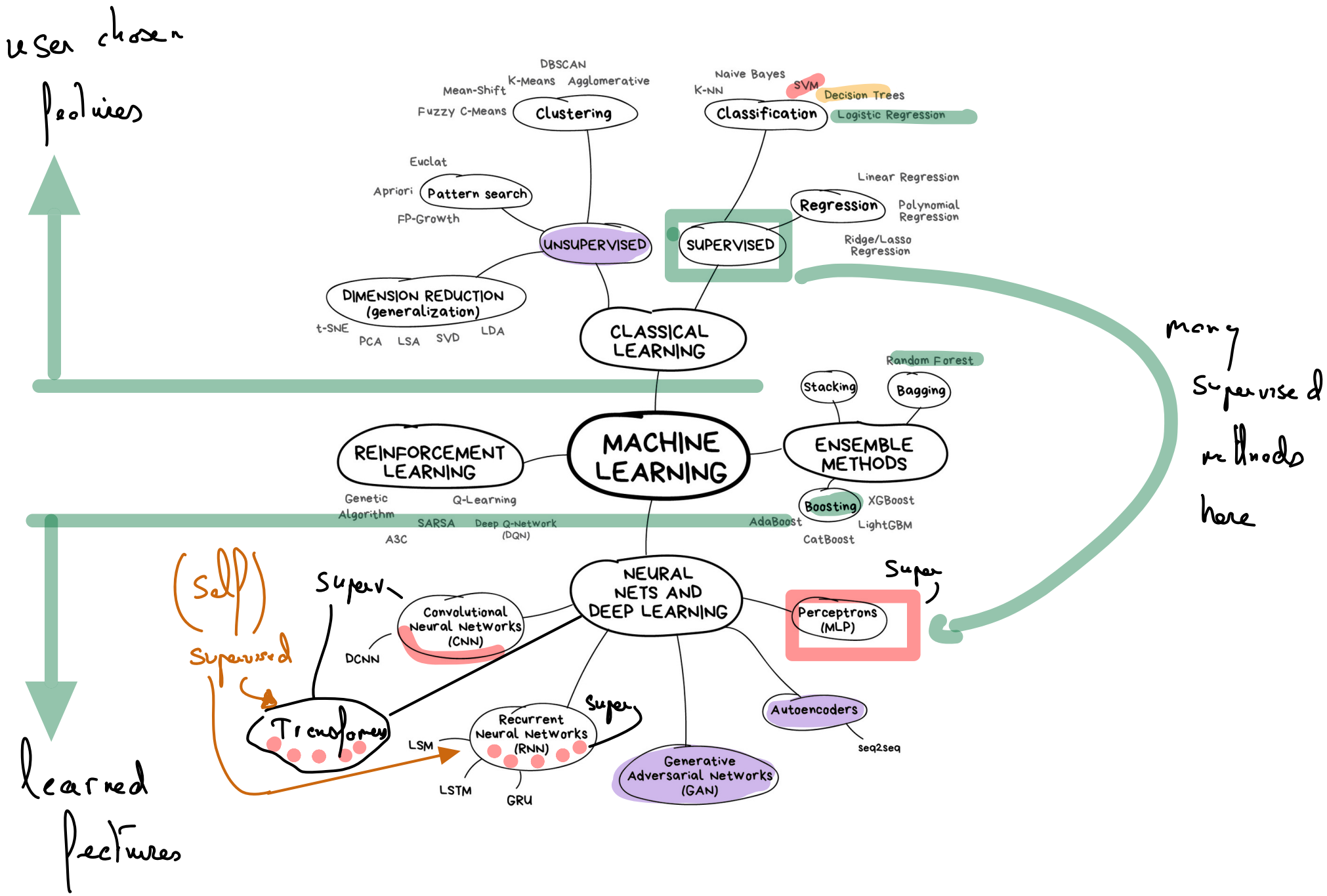
# Deep Learning in one slide + Goals of the afternoon

- **How does it work:**
  - Automatically learn representations of observations
  - Learn highly non-linear models.
- **What does it require:**
  - Large datasets with structure
  - Computational power
- **Why now:**
  - Combination of the 2 points above
  - investment !
- **Some Applications**
  - Image classification; object / face recognition
  - Self driving cars
  - Automatic Translation, Information extraction
  - Caption Generation
  - Ads, recommendation systems, etc.

**Goals:** Understanding core concepts of Deep Learning.

- When and how it works on paper (data, models, architecture)
- How to implement a simple neural network with Python.
- Overview of some of the main applications and challenges.

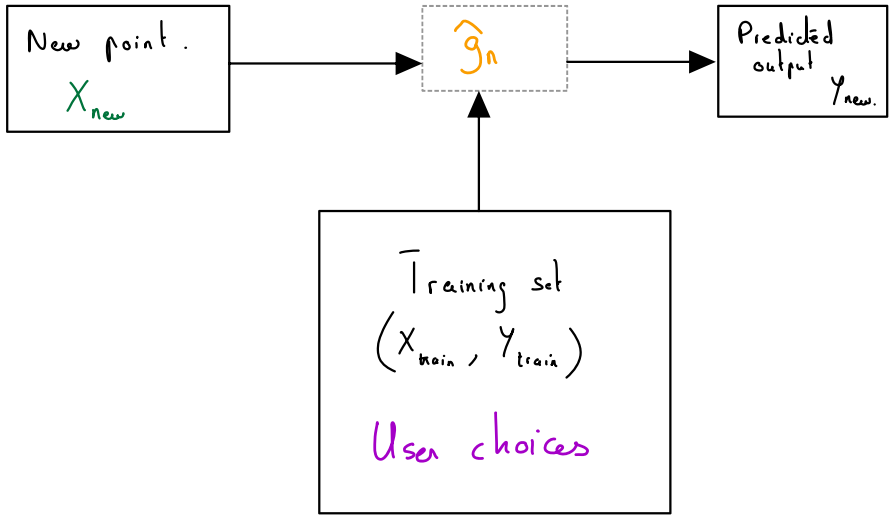
# Machine Learning



# Summary of previous lectures on Machine Learning

Important points | Inductive reasoning : Data → predictive rule
Generalize
Overfitting

1. General framework: loss, risk, generalization risk vs training risk




Loss function, Generalization Risk and data-dependent predictors

**Loss Function**

- Loss function: $\ell(y, g(x))$ quantifies the quality of the prediction $g(x)$ of $y$, e.g.:
  - 0-1 Loss (classification): $\ell(y, g(x)) = 1_{y \neq g(x)}, \quad y \in \mathcal{Y} = \{-1, 1\}$
  - Quadratic Loss (regression): $\ell(y, g(x)) = \|y - g(x)\|^2, y \in \mathbb{R}^d$

**Risk of a Decision Rule = average loss**

- Risk: $R(g) = \mathbb{E}[\ell(Y, g(x))] = \int \ell(y, g(x)) d\mathbb{P}(x, y)$, e.g.:
  - 0-1 Risk (classification): $R(g) = \mathbb{P}(Y \neq g(x))$
  - Quadratic Risk (regression): $R(g) = \mathbb{E}[|Y - g(x)|^2]$

$\mapsto$ $R$ is also referred to as *Generalization* risk, or *True* risk.

⚠ The distribution $\mathbb{P}$ is unknown.

**Data-dependent predictors**

- Training set: $\mathcal{D}_n = \{(X_1, Y_1), \ldots, (X_n, Y_n)\} \quad (i.i.d. \sim \mathbb{P})$
- Goal: build a data dependent predictor / classifier $\widehat{g}_n$ based on the training data
- That has a minimal generalization risk $R(\widehat{g}_n)$.

$R(\widehat{g}_n)$ = average loss on a "new point" $(X, Y) \sim \mathbb{P}$, independent from $D_n$

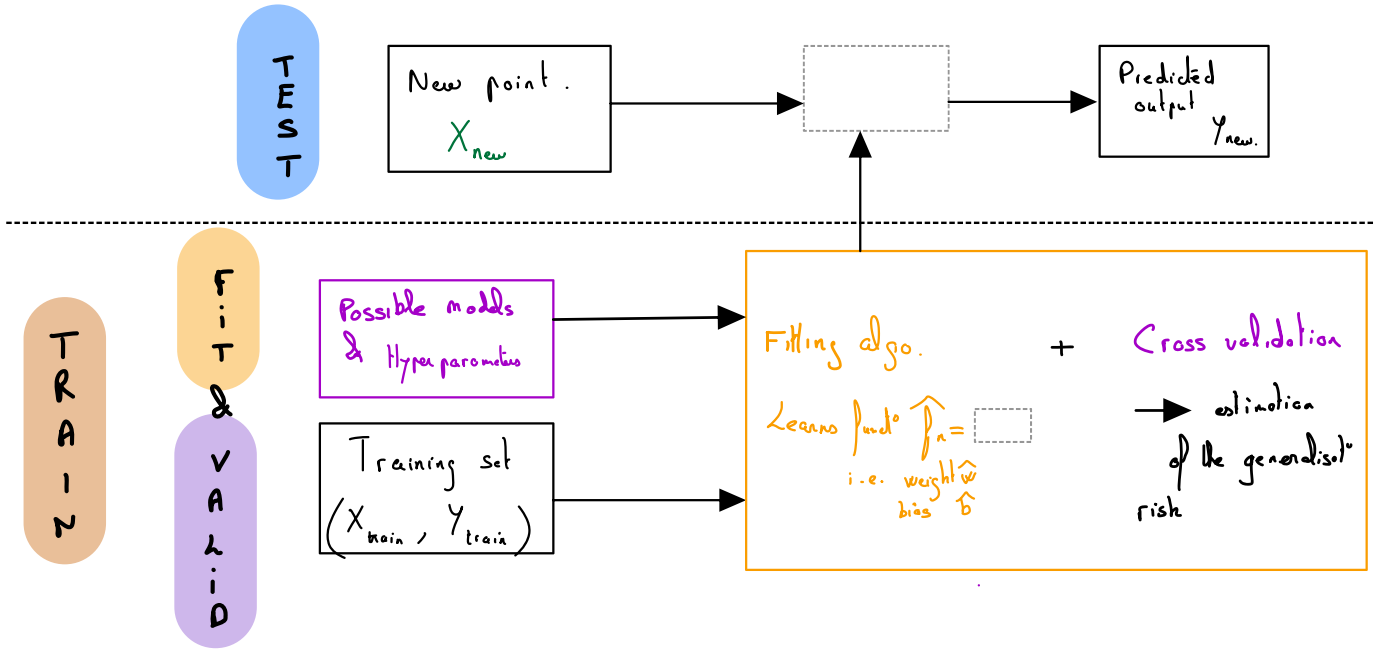
$\leftrightarrow$ Our goal is to predict well on inputs/outputs pairs that **we have not seen in the dataset**, i.e. *generalize* well.

THM 1 .

# Summary of previous lectures on Machine Learning

Key concept : Cross validation and its reason to be

1. General framework: loss, risk, generalization risk vs training risk
2. SML Pipeline
   - Train (Fit + validation) / Test / Deployment
   - Evaluation of validation scores through cross validation to pick the best method/hyperparams



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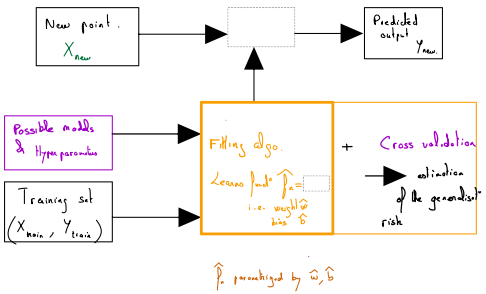
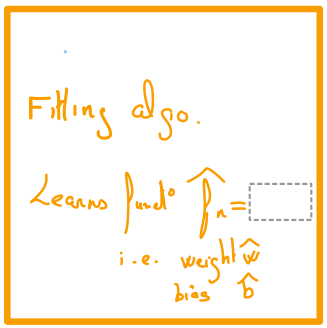
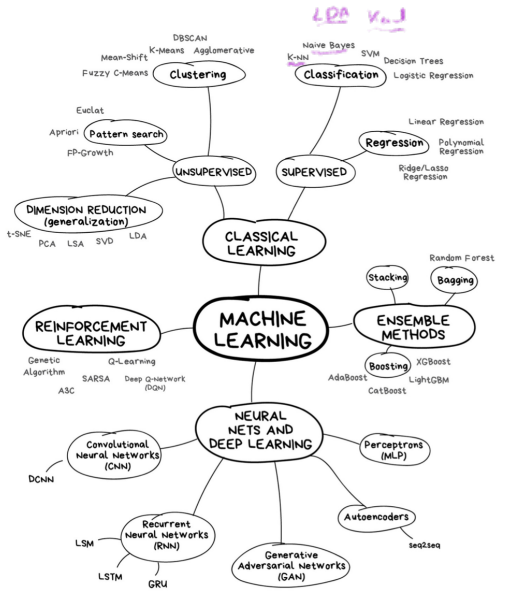
3. Methods: Linear, Logistic regression, Decision Trees, Random Forests, Boosting



# Summary of previous lectures on Machine Learning

1. General framework: loss, risk, generalization risk vs training risk
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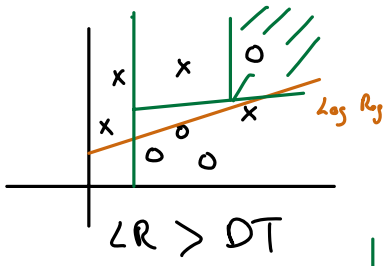
3. Methods: Linear, Logistic regression, Decision Trees, Random Forests, Boosting
4. Opening the black box: for each method
   - How to predict ( $\leftrightarrow$ interpretability)
   - How fit is made ($\leftrightarrow$ speed, complexity) [Computer learning parameters]
   - Important Hyper-parameters [User chosen]
   - Drawbacks and Strengths
   - Underfitting, overfitting risks ($\leftrightarrow$ Performance)
     $\rightarrow$ regularization techniques (avoiding overfitting by modifying the method)

$f_n =$ [ ] : Lin. Reg | Polynomial Linear reg | Logistic Reg | Dec Trees | RF/Boosting

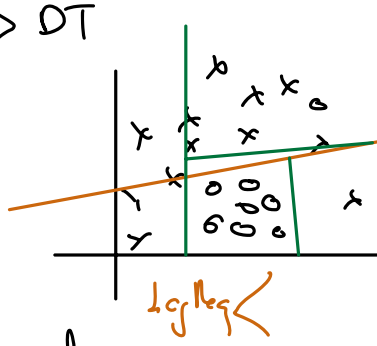
| | Lin. Reg | Polynomial Linear reg | Logistic Reg | Dec Trees | RF/Boosting |
|---|---|---|---|---|---|
| How to predict • predict | linear model. $w_1 X_1 + w_2 X_2 + \dots + w_d X_d$ | it's lin reg on polyn. features $T \mapsto 1, T, T^2, T^3$ | linear model for classif $\text{sign}(w^T X)$ | Yes/No | |
| How to fit • fit | incremental improvement SGD (or closed form formula) → | → | II | one split by one split | not II ≠ |
| Key hyperparameters | none; yes if regularized (sparsity) LASSO → | → | same as LR | depth ... | # trees subsampling diversity |
| ⊕ | simplicity / speed stability / (interpret) → | more powerful (non linear rel) | same as LR | speed non linear interpret | more stable less overfitting! |
| ⊖ | Accuracy (linear relat) | → | so LR | overfitting | less interp slower. |
| Overfitting / Underfitting → regularization technique | both: often underfit possibly overfit if d very large | made higher d: 80 → 6400 → overfitting risk | so LR | yes | not too much |
| Approach | ERM. | ERM | ERM | not ERM | not ERM |

e.g. House Price prediction dataset, with $d = 80$ polynomial features of degree 2 create more or less $80^2 = 6400$ features → leads to Boosting.

Which is the best model?




→ no definite answer




* even for a given data distribution IP, the ability of a model to perform well depends on the nbr of observations.

* The best model depends on IP (and n)

* The best model depends on Business perspective (loss is, good, Speed / Interpret / Accuracy)

# Outline

# Outline

# From logistic regression to Multi Layer Perceptron 1

ERM

Supervised ML problem:

1. Data $D_n = (X_i, Y_i)_{i=1,\dots n}, X_i \in \mathbb{R}^d, \quad Y_i \in \{-1, 1\}$
2. Goal: Predict a the label for a new point.

**Empirical risk minimization (Discriminative approach)**

- No statistical model!
- ~~Define~~ Choose a loss function $\ell$
- Choose a function class $\mathscr{F}$


- Goal:

$$\min_{f \in \mathscr{F}} \mathbb{E}[\ell(f(X), Y)]$$

- ERM

$$\min_{f \in \mathscr{F}} \frac{1}{n} \sum_{i=1}^{n} [\ell(f(X_i), Y_i)]$$

we minimize the Empirical risk.

loss

$\ell(Y_i, f_\omega(X_i))$

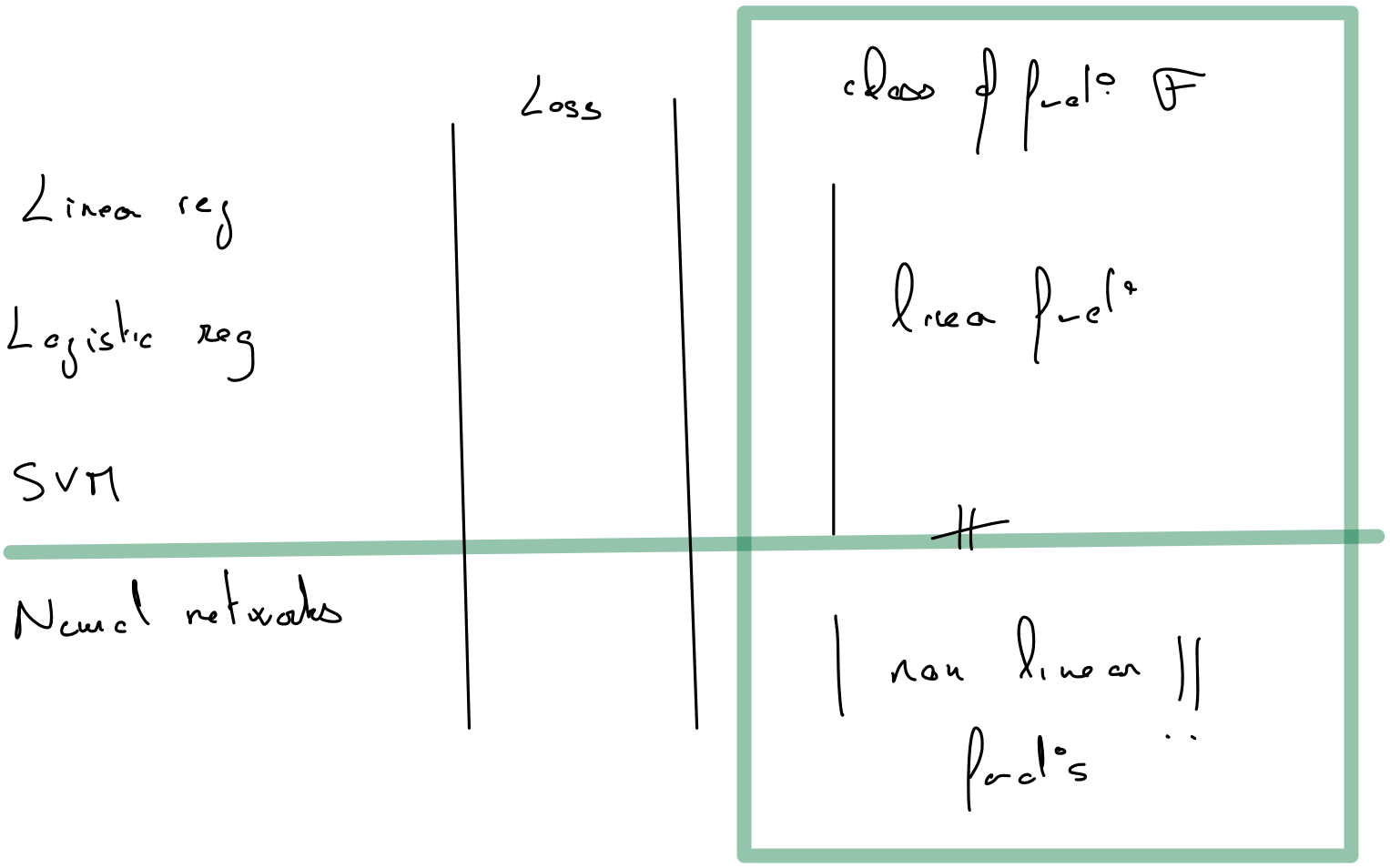
$\frac{1}{n} \sum_{i=1}^{n} \ell(Y_i, f_\omega(X_i))$

average loss on fit dataset.

→ empirical risk

def.

↳ we have an opt pb → we use an opt algo to find a solution = SGD "II"

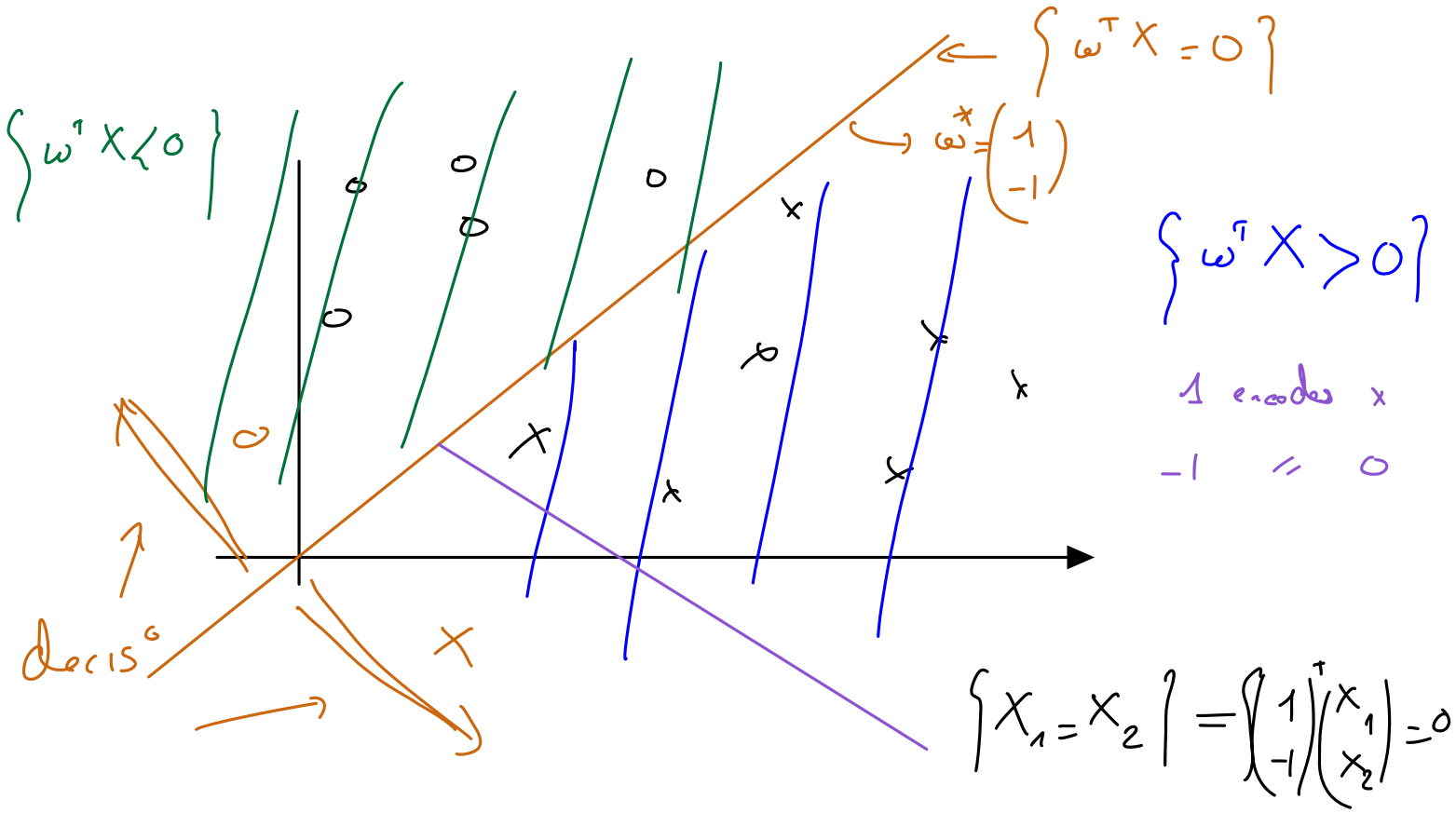

Loss
class of func F
Linear reg
Logistic reg
SVM
Neural networks
linear funcs
non linear funcs

# Linear models

Class of *soft* linear separators:

$$\mathscr{F}_{lin} = \{g_w : X \mapsto w^\top X \quad , w \in \mathbb{R}^d\}$$

Binary prediction from a soft predictor: $X \mapsto \text{sign}(w^\top X)$



# Linear models

Class of *soft* linear separators:

$$\mathcal{F}_{lin} = \{g_w : X \mapsto w^\top X \quad , w \in \mathbb{R}^d\}$$

01 loss $\hookrightarrow$ accuracy

Binary prediction from a soft predictor: $X \mapsto \text{sign}(w^\top X)$

truth my prediction

**01 loss**:

$$\frac{1}{n}\sum_{i=1}^{n} 1_{Y_i \neq \text{sign}(w^T X_i)} = \frac{1}{n}\sum_{i=1}^{n} 1_{-Y_i w^T X_i > 0}$$

**Logistic regression loss**

$$\frac{1}{n}\sum_{i=1}^{n} \log(1+\exp(-Y_i \; w^T X_i))$$

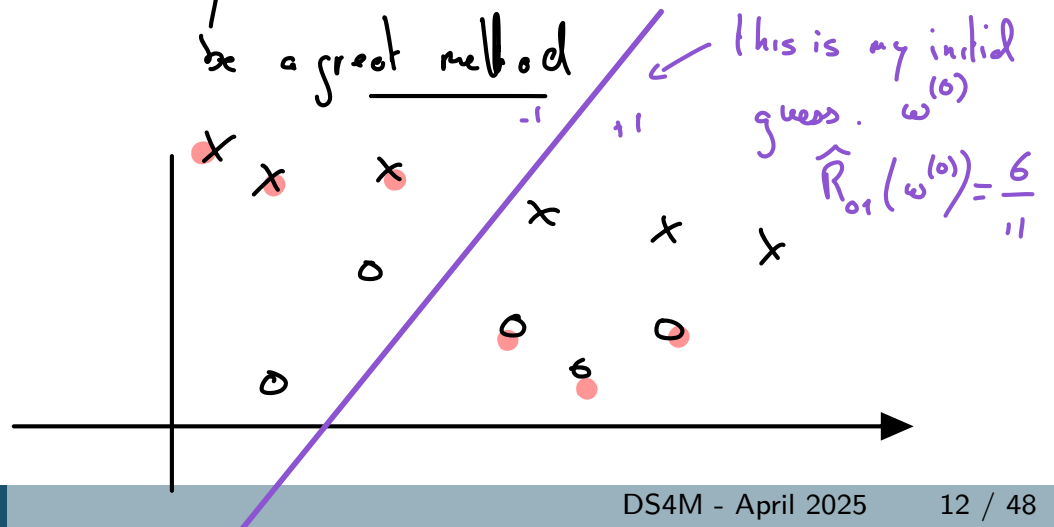
logistic loss

(Pb 1)

why not solve ERM for 01 loss

$$\min_{w} \frac{1}{n}\sum_{i=1}^{n} 1_{\{Y_i \neq \text{sign}(w^T X_i)\}}$$

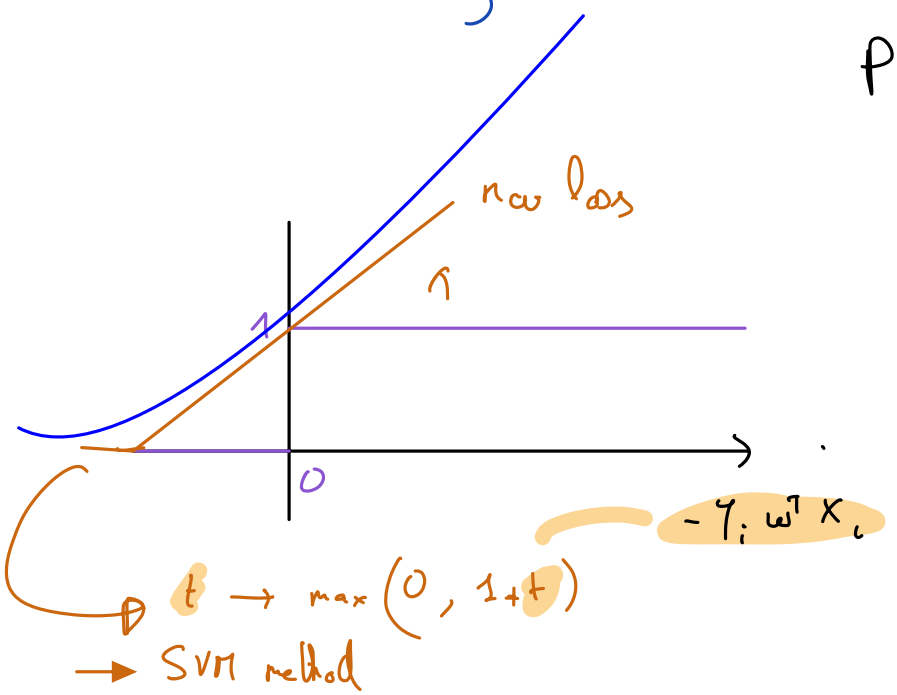
? if I could solve that, this would be a great method



01 loss :

loss of 1 if $-y_i\, w^T x_i > 0$

$t = \log(1 + e^t)$




$t \to \max(0, 1+t)$

→ SVM method

01 loss | is flat almost everywhere
non differentiable !
non cvx

Pb1 is IMPOSSIBLE to solve algorithmically (or exponential only in d).

Logistic regress° summary

* I want to use linear fnct°

* The 01 loss is { bad / impossible } to optimize

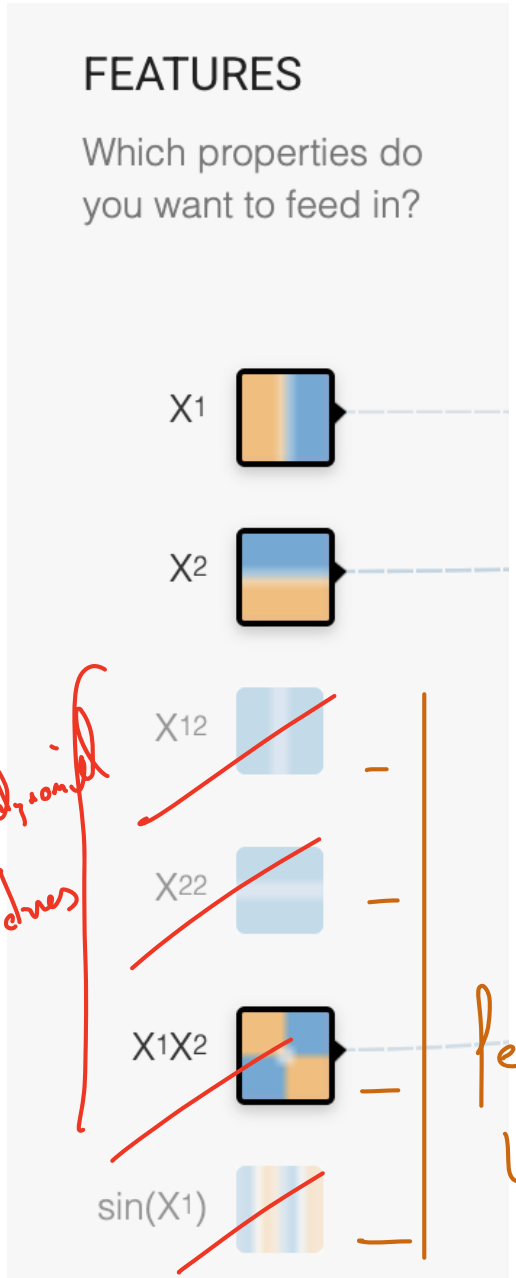
* Change the 01 loss to logistic loss

→ get logistic regress° !

# Outline

# Let's NN playground: Link

**1. Understand what the features are. Understand what colors represent.**




features I use.

$X^1$ → the x axis value

$X^2$ → the y axis value.

Whenever you hover over a square

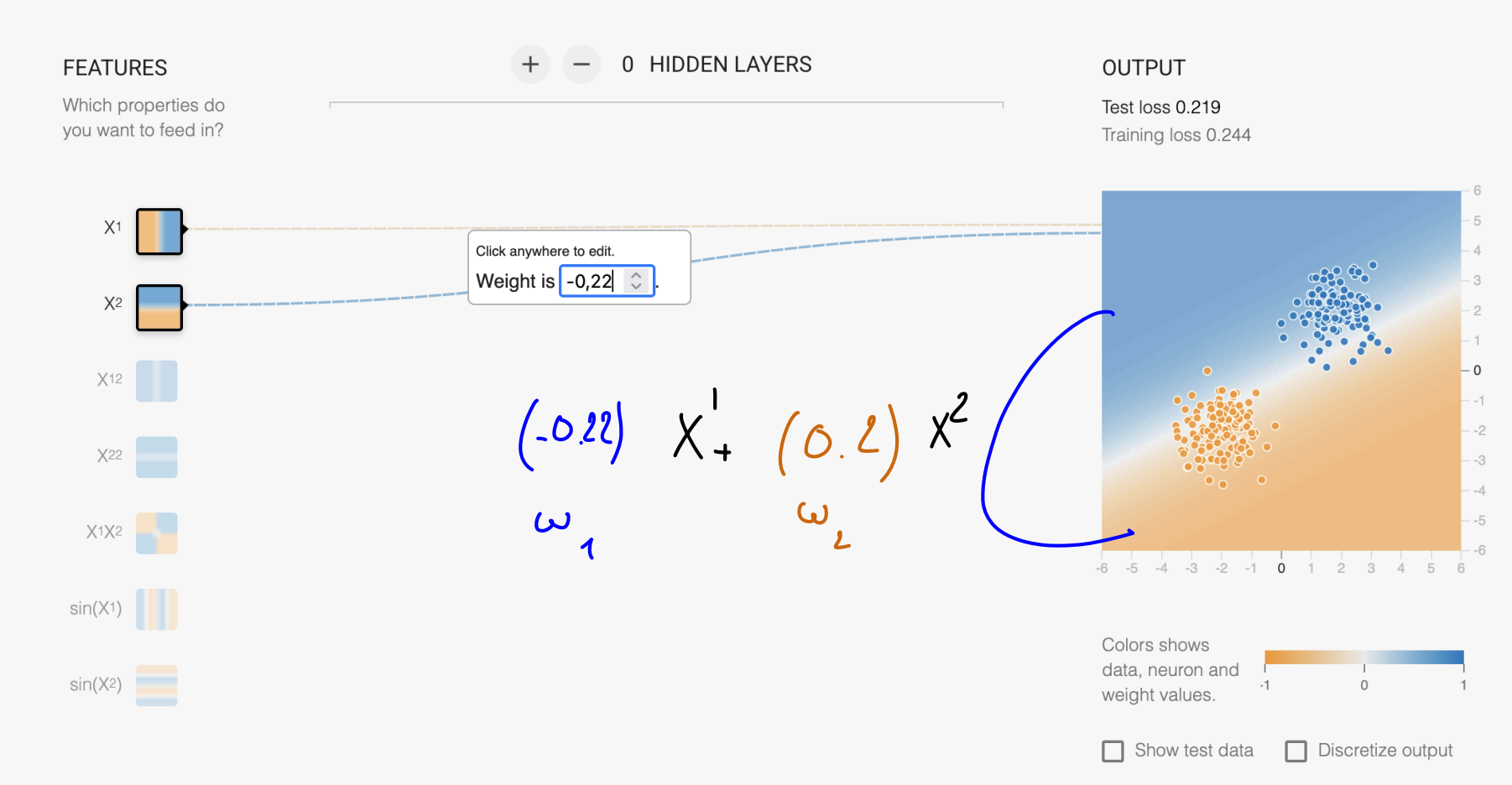
I am restricting myself to $X^1 X^2$

polynomial features

features that I could pick.

# Let's NN playground: Link

**2. Consider the simplest dataset, an 0 hidden layer. Manually pick weights to obtain a good classification.**




What model does this correspond to? How many weights does it have?

# Let's NN playground: Link

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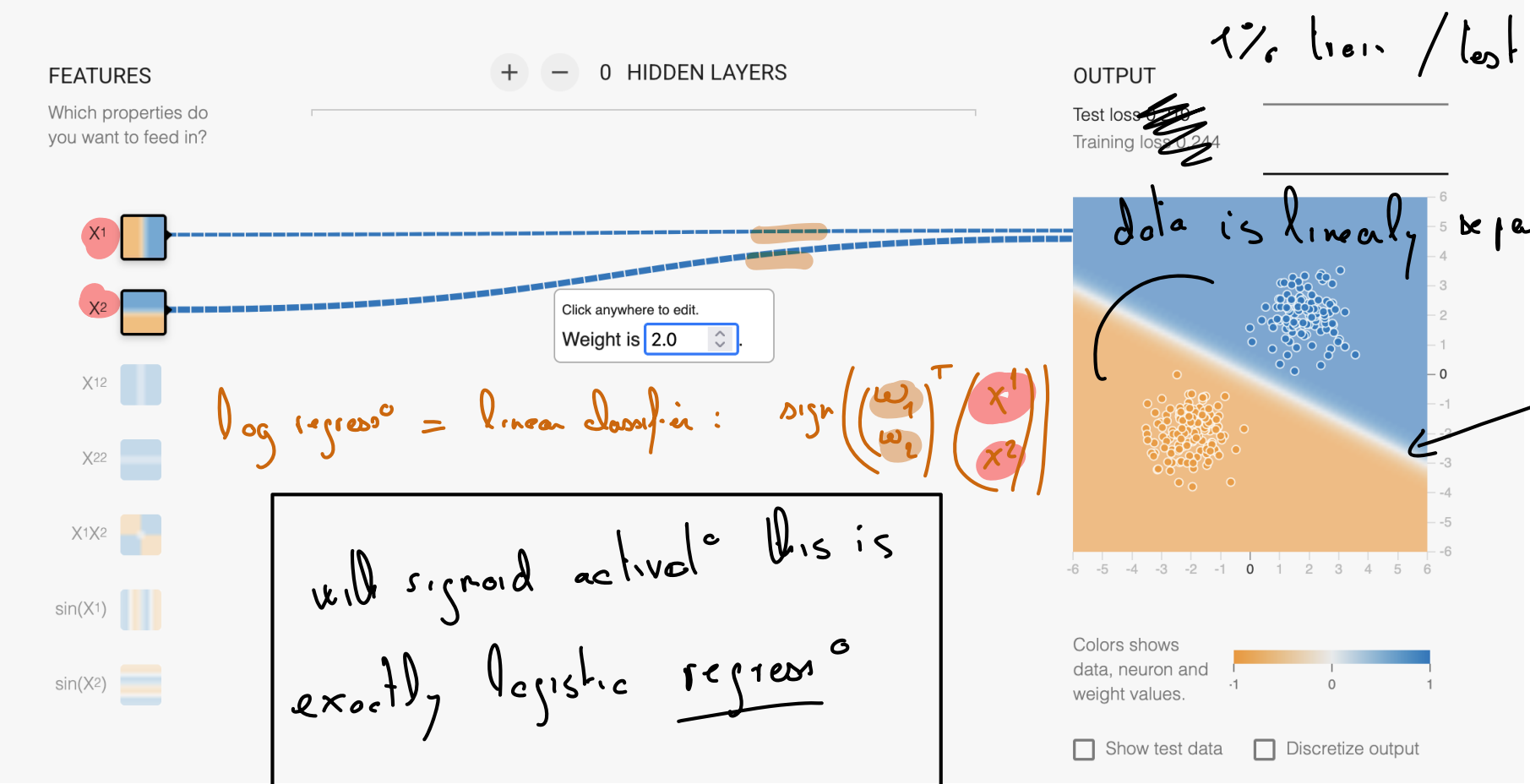



What model does this correspond to? How many weights does it have?

↳ (2)

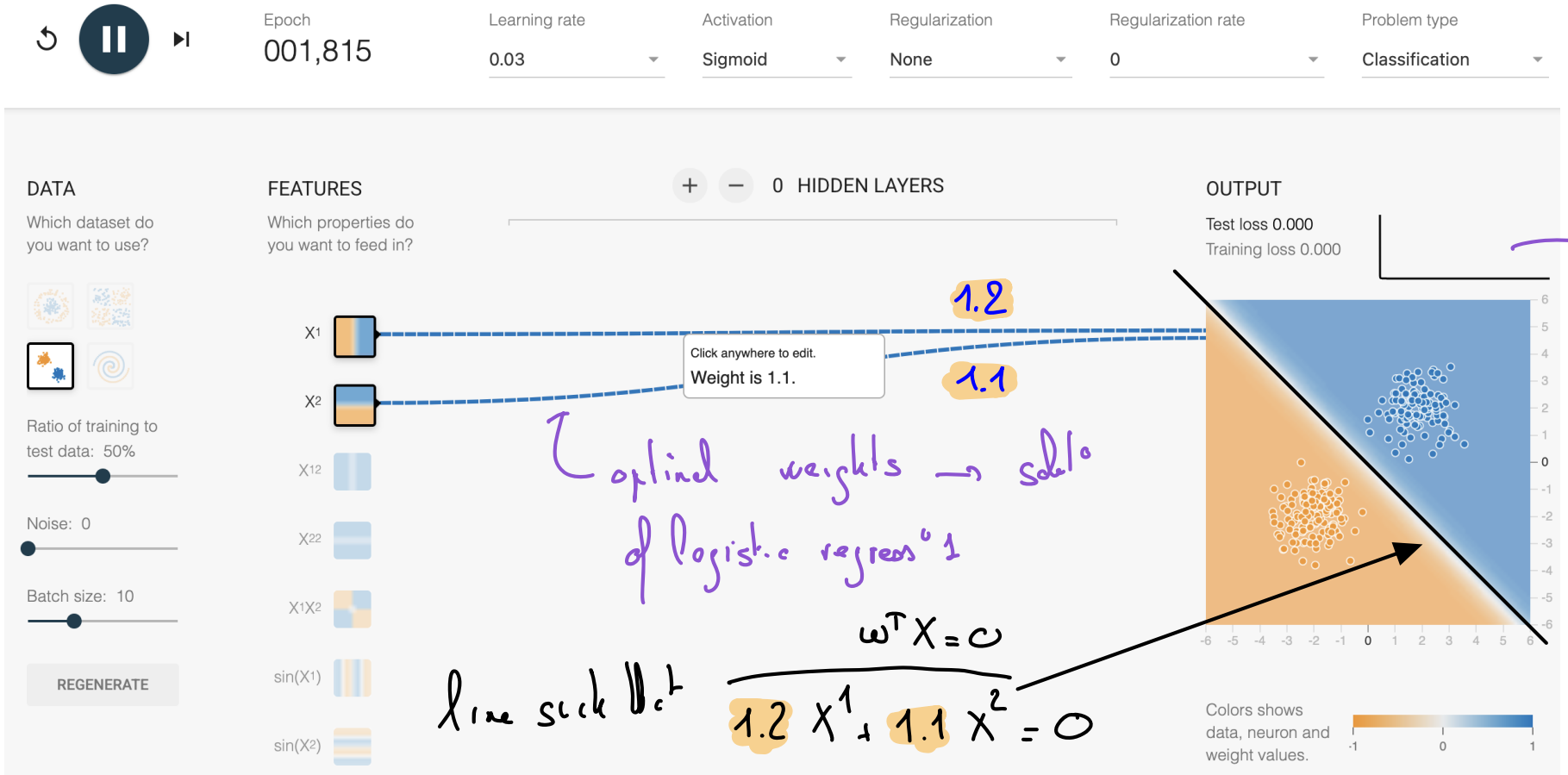
# Let's NN playground: Link

**3. Consider the simplest dataset, an 0 hidden layer. Instead of manually tuning the weights, obtain them by fitting!**


What algorithm are we running? → SGD
What can you say about the loss? →
Rerun the algorithm? What happens?

# Let's NN playground: Link

**3. Consider the simplest dataset, an 0 hidden layer. Instead of manually tuning the weights, obtain them by fitting!**



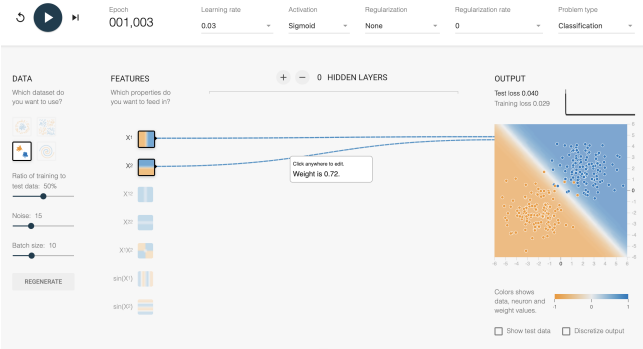

What algorithm are we running?
What can you say about the loss?
Rerun the algorithm? What happens?

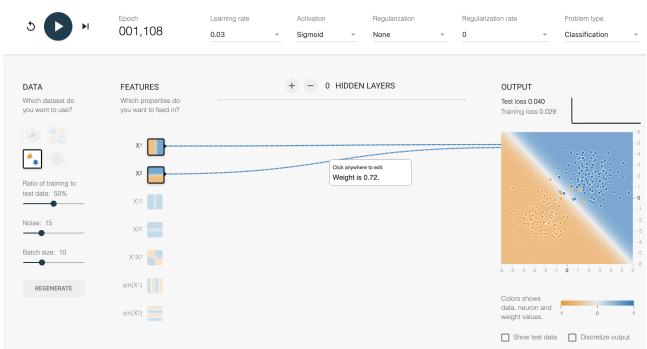
loss, train and test!

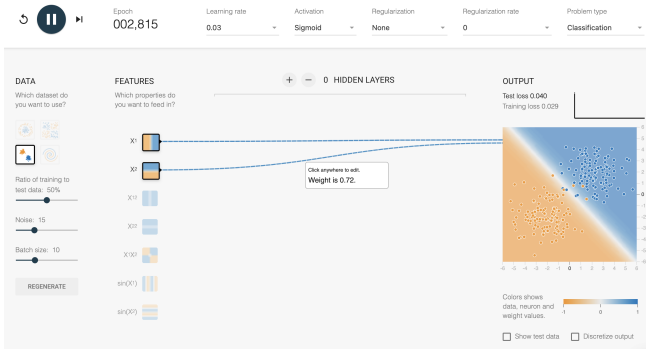
almost the same boundary — almost the same weights

# Let's NN playground: Link

**4. Increase the data noise to 15. Is the data linearly separable then? Run 3 times for 1000 epochs, compare the weights obtained**




What do you conclude / confirm on logistic regression?

# Let's NN playground: Link

## 5. Change the dataset to  . Run logistic regression. What do you observe?


worst possible error

underfitting

50% error

why:

Log Reg: linear separ° !

→ there is no linear separ° here!!

What do you suggest?

Sol°s: polynomial feature → S weights, ranks.

# Let's NN playground: Link

**6. Add a single hidden layer, first with a single neuron, then with 4 neurons.**
Keep only features $X^1$ and $X^2$.

↳ Single neuron : not enough

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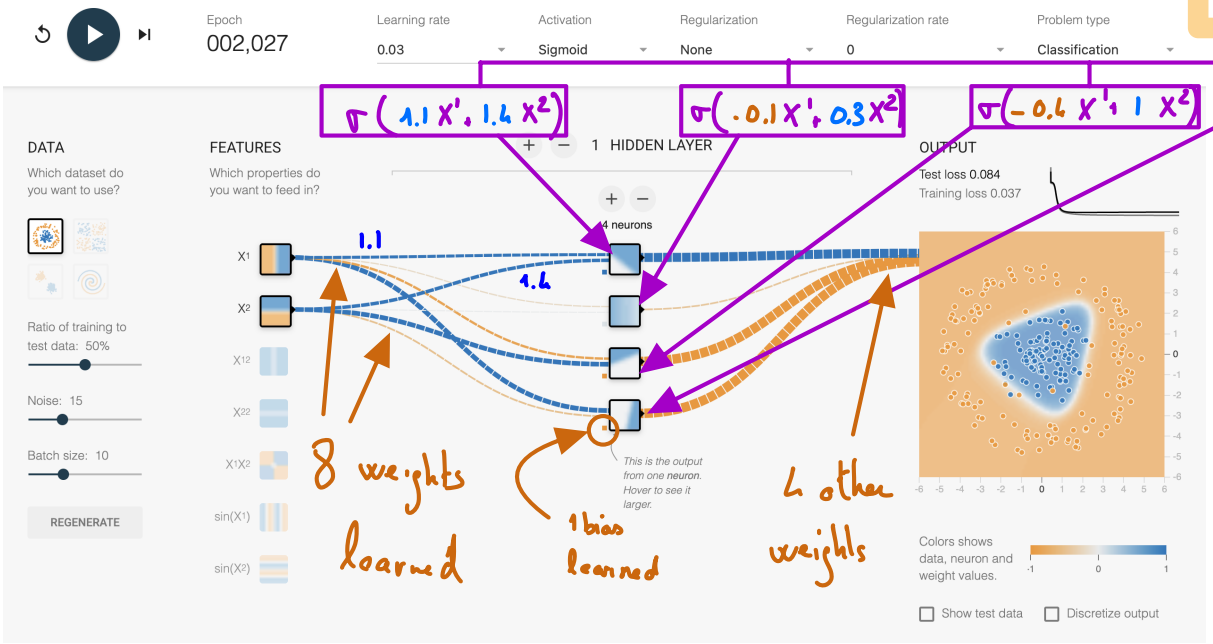


♡ This is your first NN

This is feature learning!
4 features $w_1 X^1 + w_2 X^2$ are learned, that can be combined into the output!

What do you observe? → it works !! → Loss gets to 5-8% on the test
What have we learned? → weights ! 16 : 12 weights + 4 biases
How does that illustrate *feature learning*?
What happens if you restart learning? → completely unstable / completely ≠ after restart.

# Let's NN playground: Link

**6. Switch to the more complex spiral dataset**  **. Increase the number of neurons. Fit the model.**
Keep only features $X^1$ and $X^2$.

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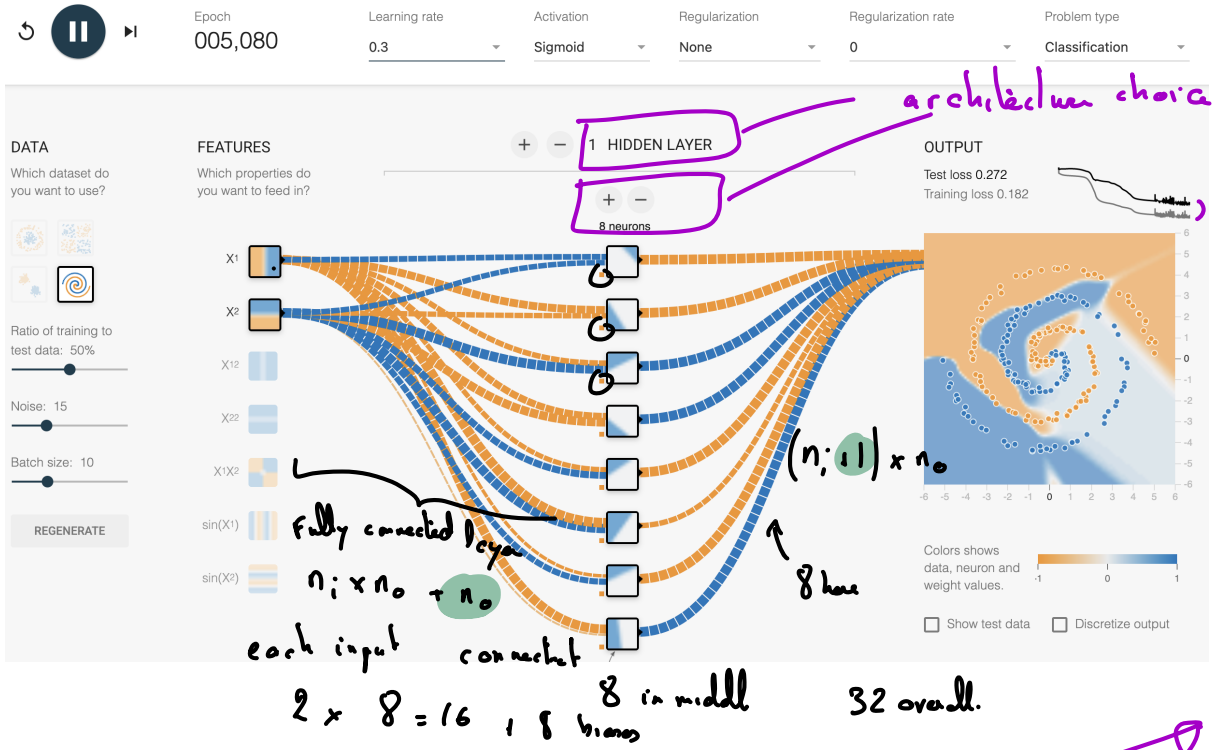


What happens? How stable is the training?

How many weights are you learning now? Are there biases? What are those?

# Let's NN playground: Link

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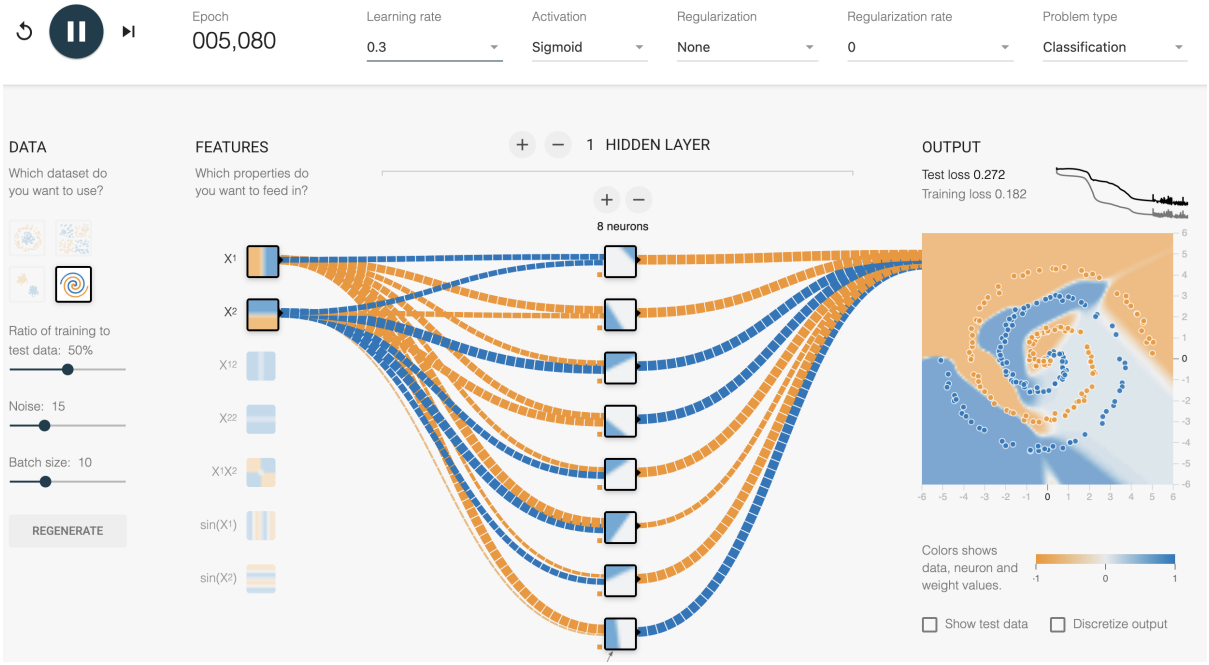
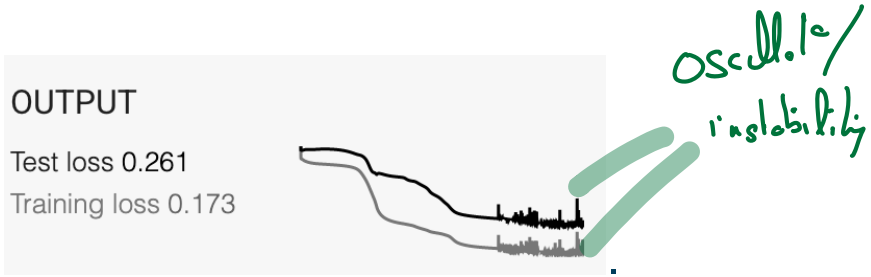



What can you say about the algorithm dynamic ?



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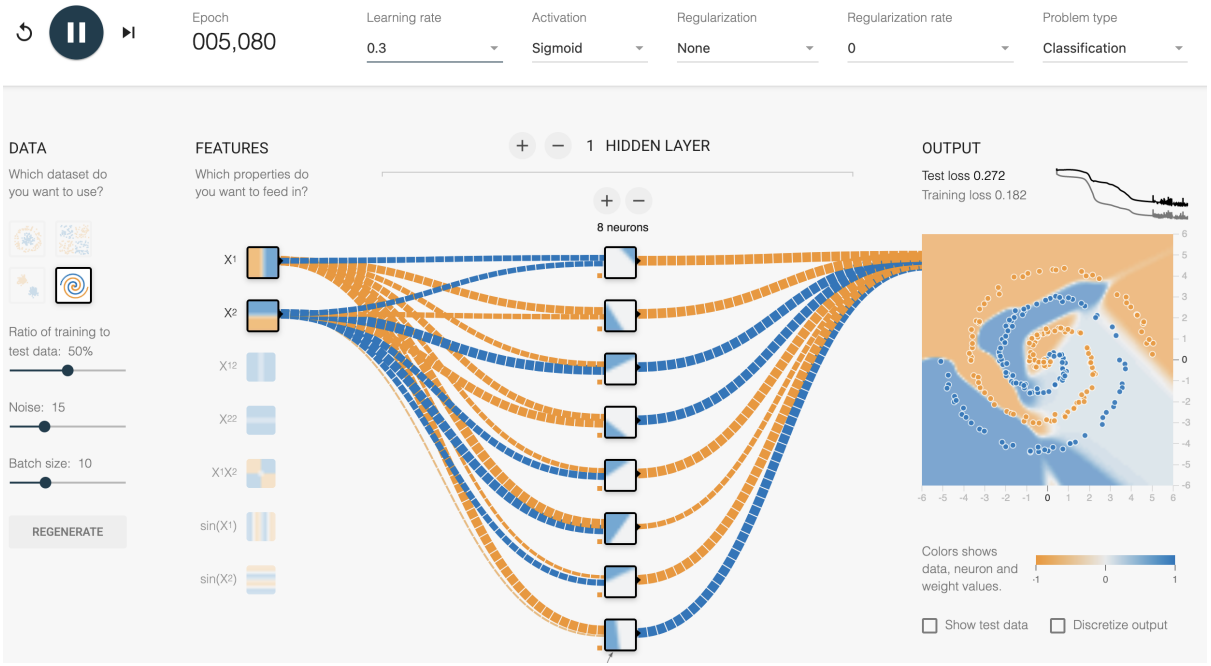



Test loss will

8 neurons on 6 layer

Relu act

step size 0.03

→ 2% of test loss

What do you suggest next? → increase the number of layers

# Let's NN playground: Link

**7. Explore configurations : what do you obtain?**

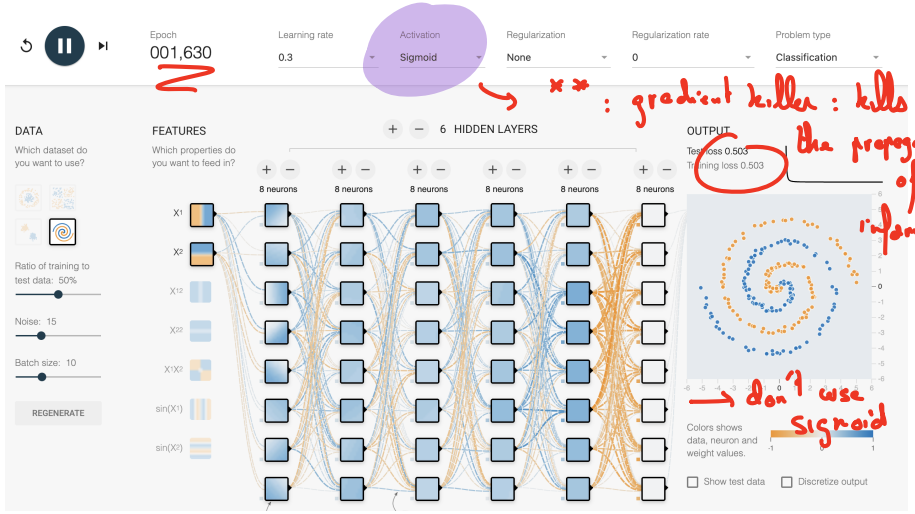
# Let's NN playground: Link

**7. Explore configurations : what do you obtain?** What is the role of the activation function? What happens for a deep architecture for Linear / Sigmoid / Relu / Tanh?

# Let's NN playground: Link

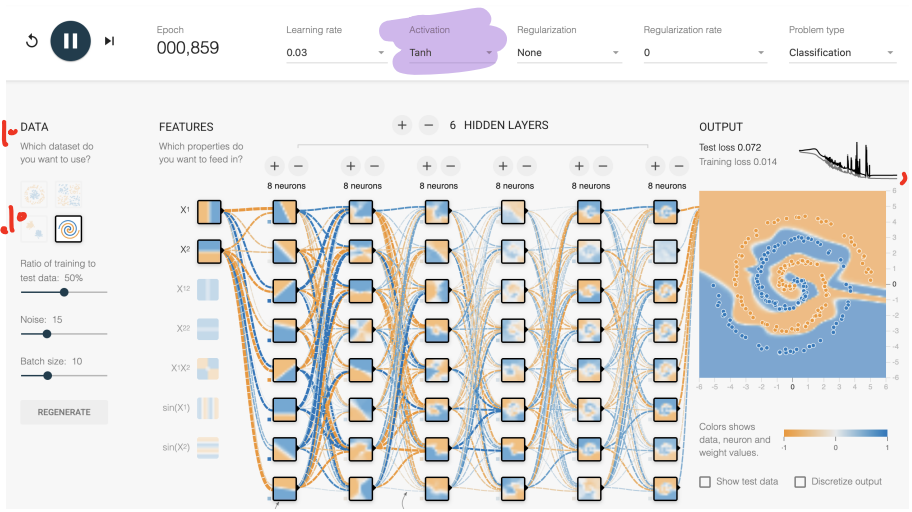
hyperparam

**7. Explore configurations : what do you obtain?** What is the role of the activation function? What happens for a deep architecture for Linear / Sigmoid / Relu / Tanh?

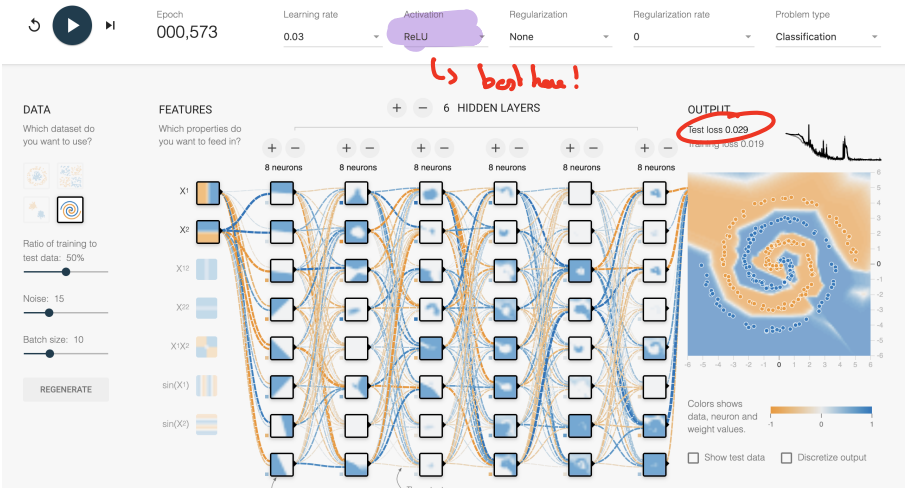



** : gradient killer : kills the propagation of information

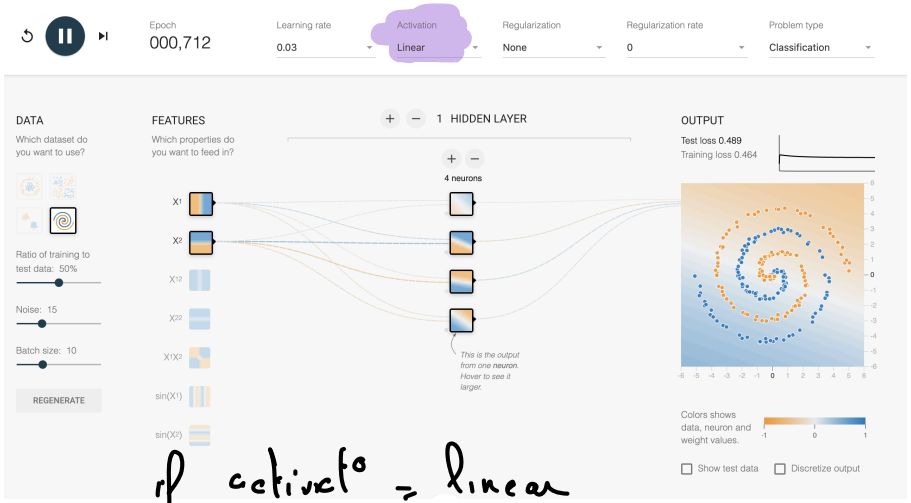
don't use sigmoid




ouf!




↳ best here!




Activation = Linearity breaker

if activ° = linear
the input → output relation remains linear! ∴

never use linear!!

# Let's NN playground: Link

**8. How do the features learned evolve depending on the layer?**

# Let's NN playground: Link

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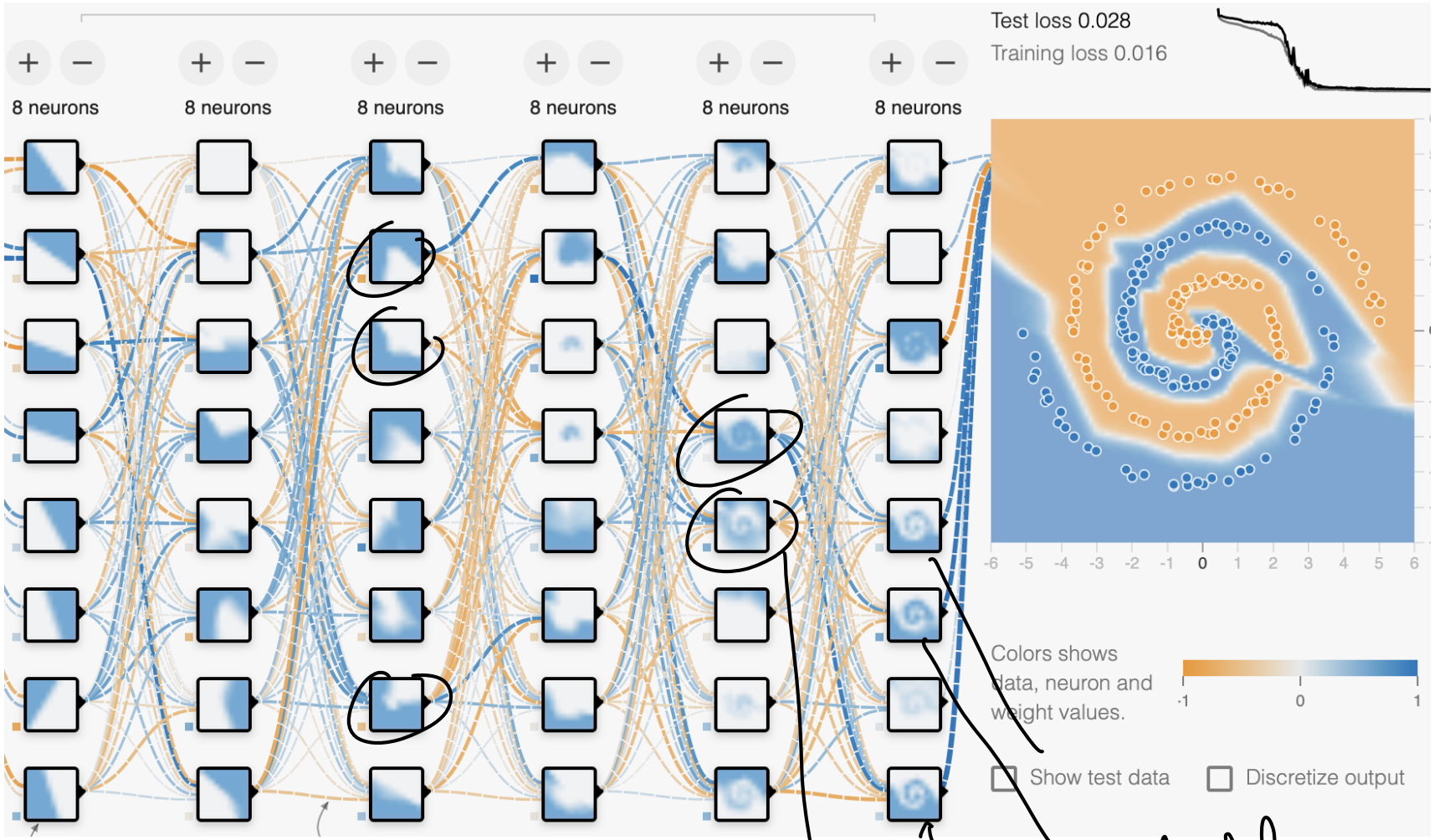
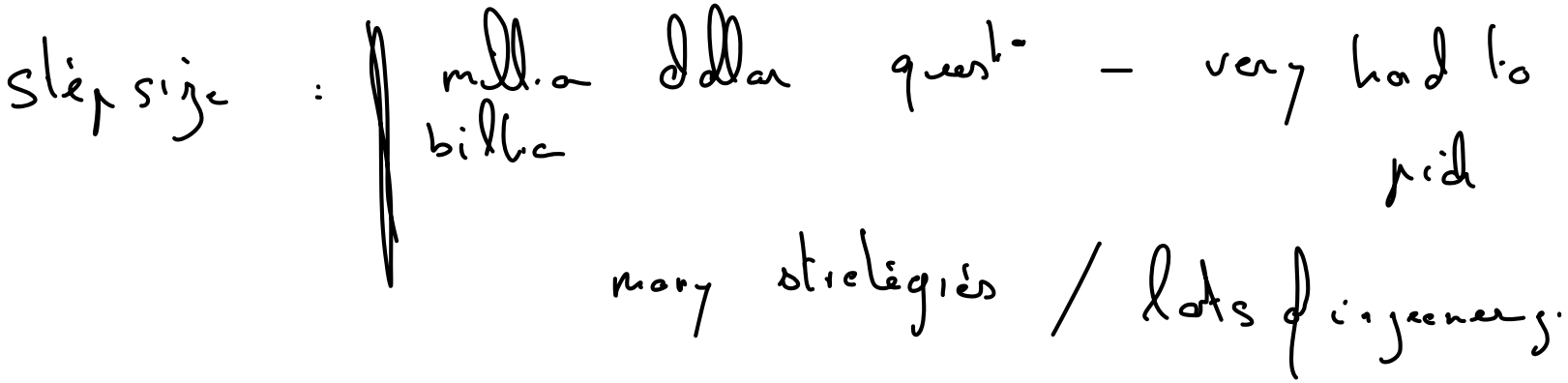


# Let's NN playground: Link

**9. What are the hyper-parameters of the SGD algorithm? Play with them.**




Advanced

1. What happens if you initialize all weights to 0?

# Conclusion on NNPG

1. "Architecture":
   - * neuron
   - * hidden layer
   - * activation
   - * SGD looks like

   learned
   - * weights
   - * biases

   ...

2. What we have seen :
   - Feature learning
   - Implicit bias - randomness in the process
   - Flat moments then progress
   - Impact of depth.

3. *Protip* : do it again in one year !!
   → add it in your calendar

# Wooclap time!

1 Allez sur wooclap.com

2 Entrez le code d'événement dans le bandeau supérieur

Code d'événement
**BLCOFS**

1 Envoyez @BLCOFS au 06 44 60 96 62

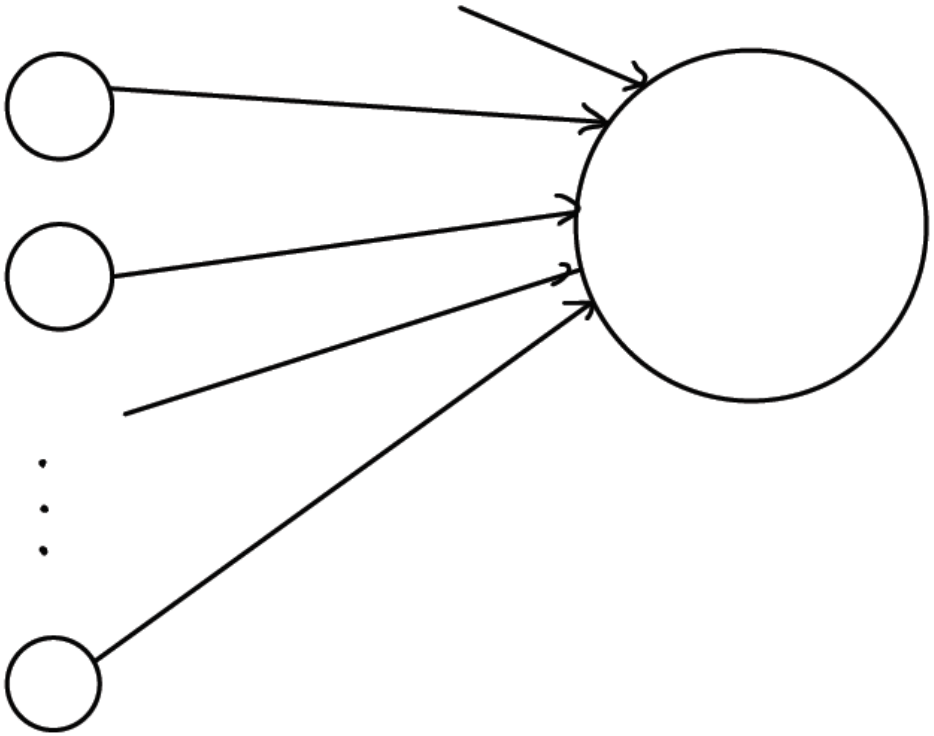
2 Vous pouvez participer

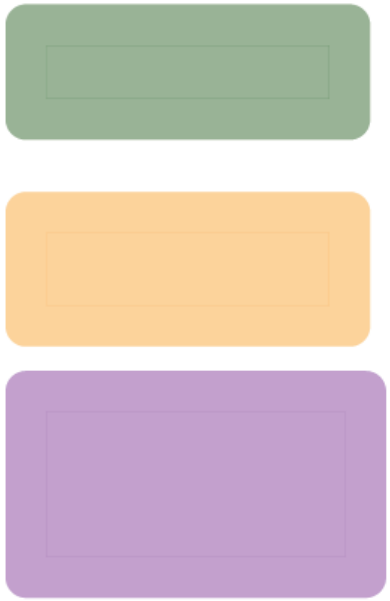
Désactiver les réponses par SMS

# Outline

# From logistic regression to Multi Layer Perceptron 3

- Class of functions for logistic regression: $\mathscr{F} = \{X \mapsto w^T X,\ w \in \mathbb{R}^d\}$.
- Prediction for a new point is $1_{\sigma(X_i)>1/2}$
- Can be seen as predicting a probability $\sigma(X_i)$ that the output is 1





This is a neural network, with one neuron !

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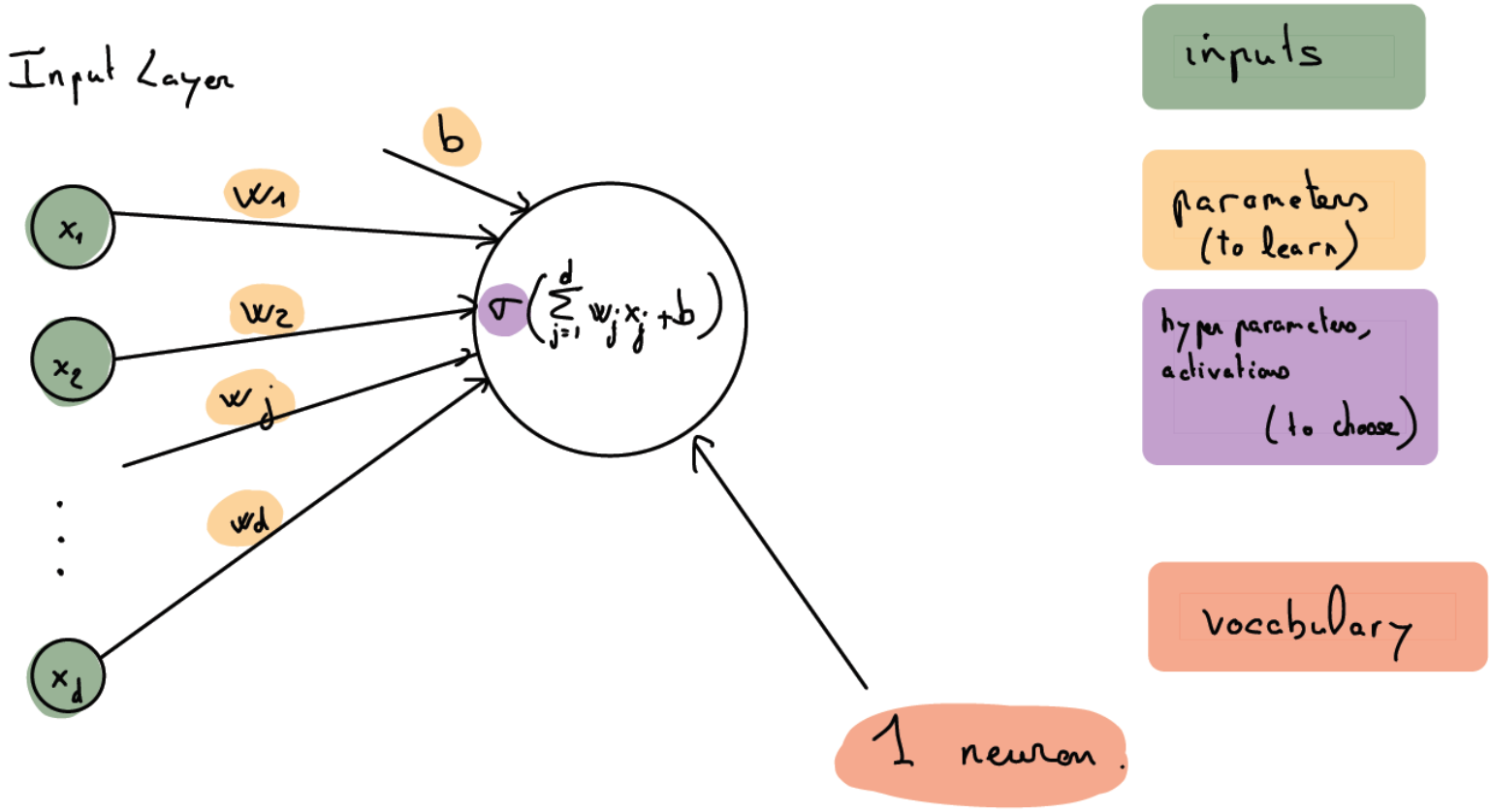
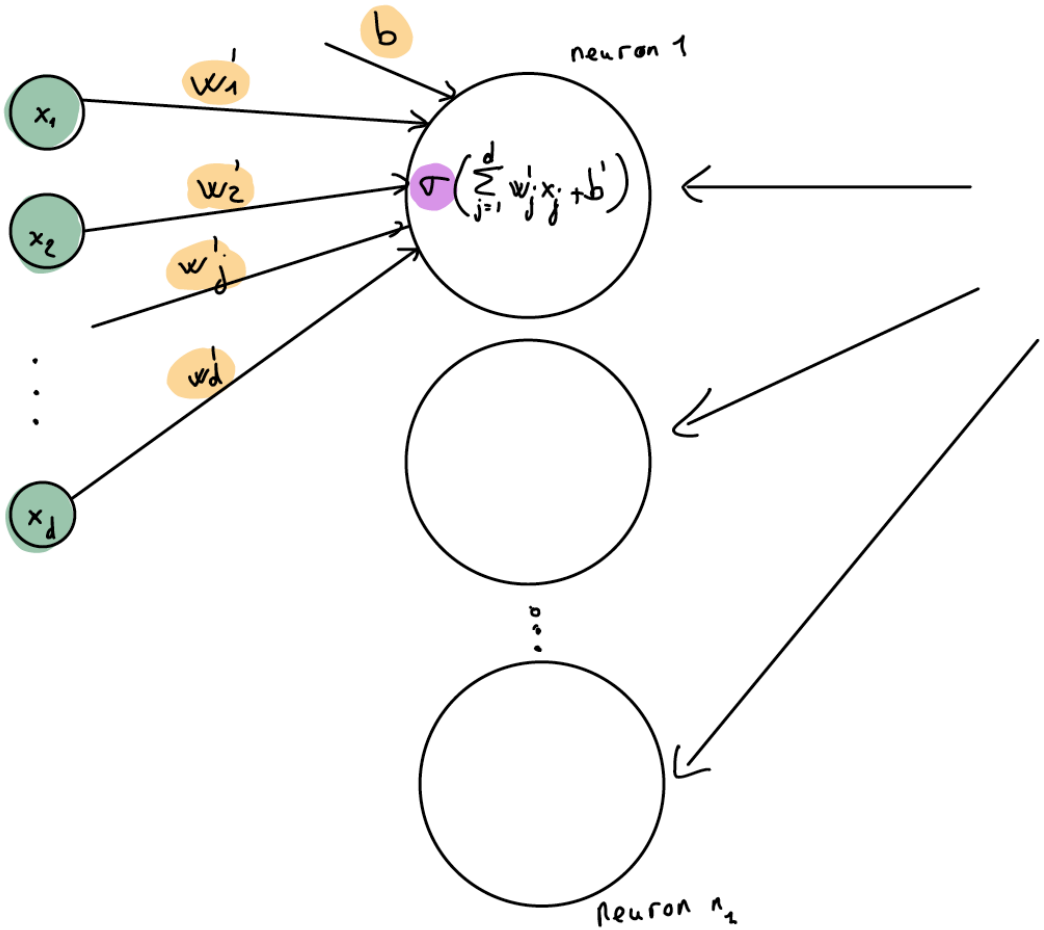



This is a neural network, with one neuron ! = This is logistic regression

# From logistic regression to Multi Layer Perceptron 4

Extend to

- Multiple Neurons
- Multiple Layers



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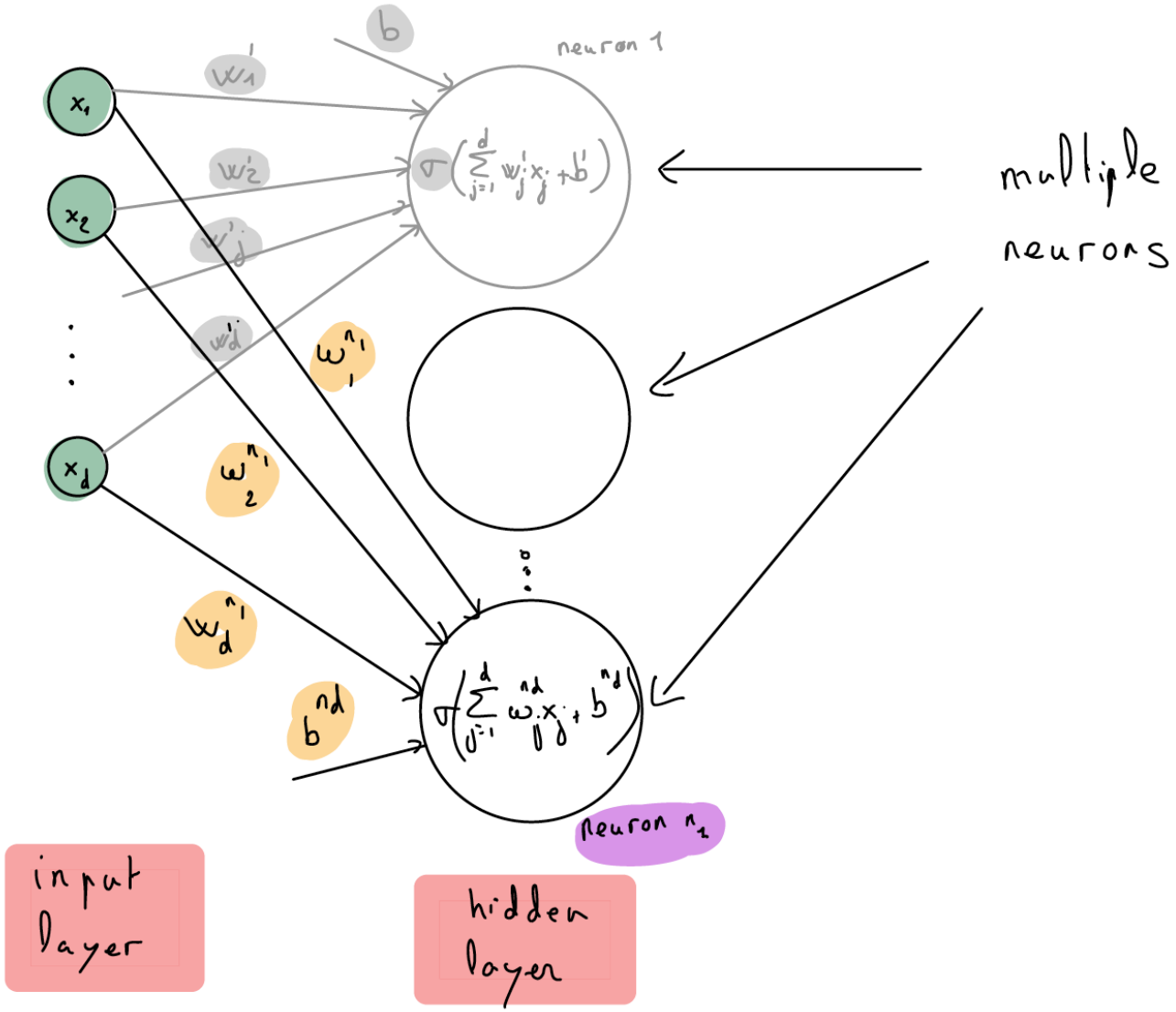
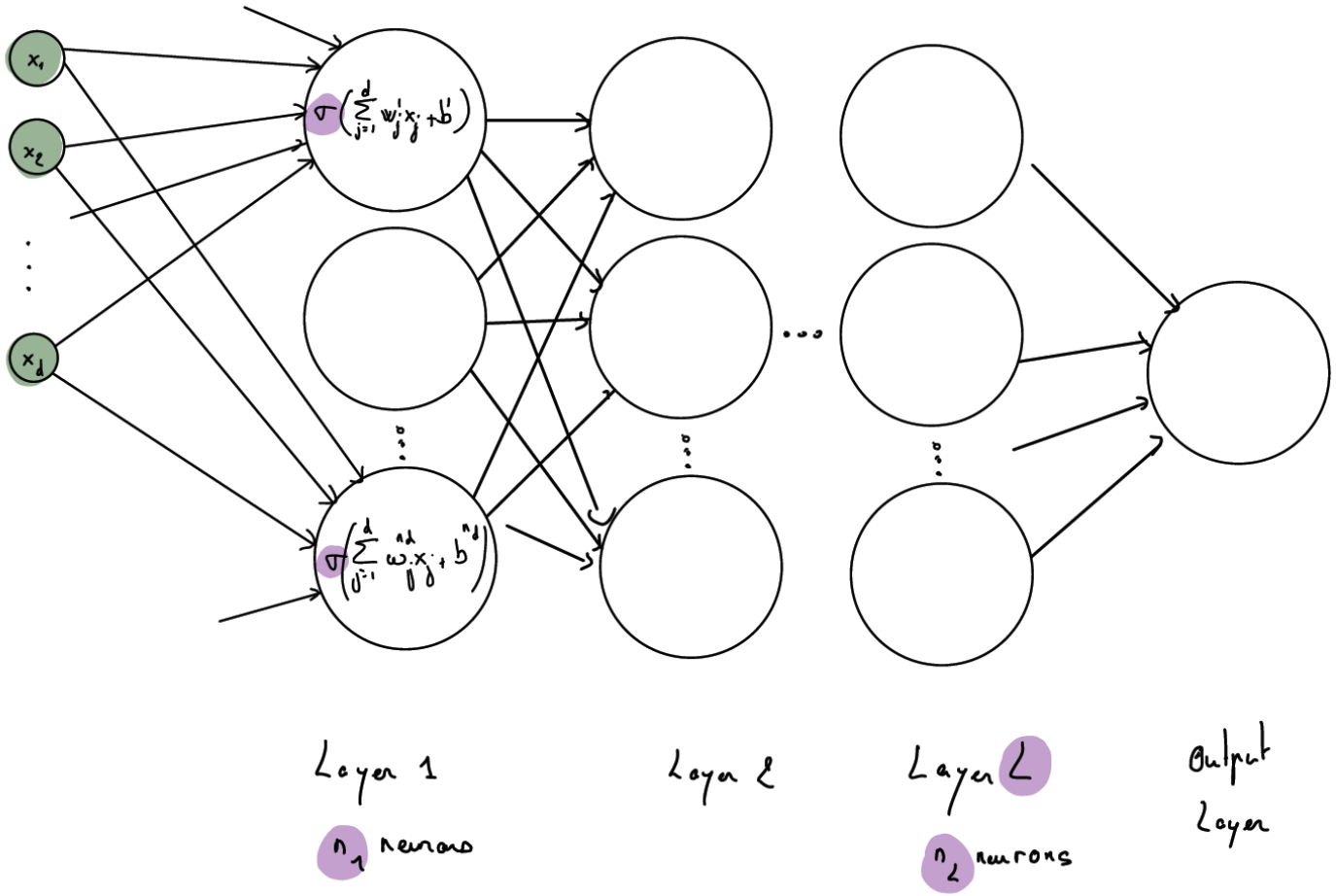


# From logistic regression to Multi Layer Perceptron 5

Extend to

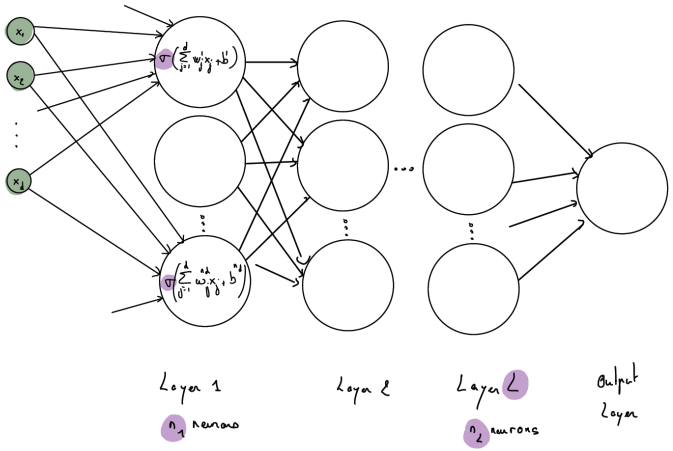
- Multiple Neurons
- Multiple Layers



# From logistic regression to Multi Layer Perceptron 5

Neural network:

- Goal $\min_{f \in \mathscr{F}_{\text{MLP}}} \frac{1}{n} \sum_{i=1}^{n} [\ell(f(X_i), Y_i)]$
- Class of non linear functions.
  $\mathscr{F}_{\text{MLP}} = \{f(X; W, b) = \sigma(b_n + W_n \sigma(\ldots(b_1 + W_1 x))),$
  $W \in \mathbb{R}^{\cdots}, b \in \mathbb{R}^{\cdots}\}$




math. formula of the class of funct we've looked at !

Summary: NN generalize regression by looking for candidates in a much larger class of functions than linear ones.

| What needs to be learned | What needs to be chosen | Vocabulary |
|---|---|---|
| W | archit L ayers | Neuron, SGD, |
| b | Neuron | etc |
| | activate | |

# From logistic regression to Multi Layer Perceptron 5

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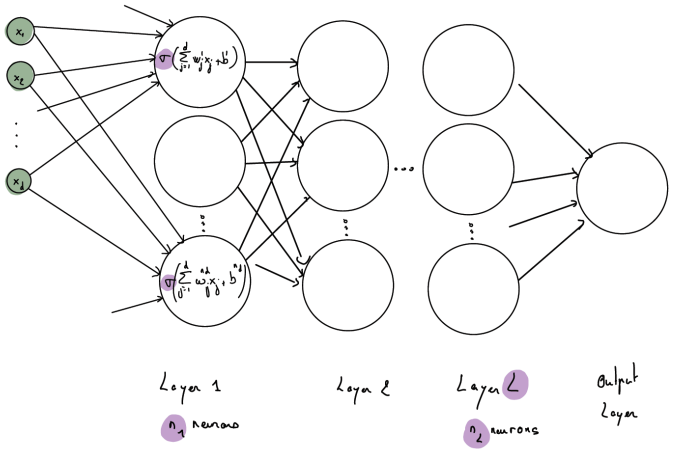



Summary: NN generalize regression by looking for candidates in a much larger class of functions than linear ones.

What needs to be learned

Parameters $W, b$

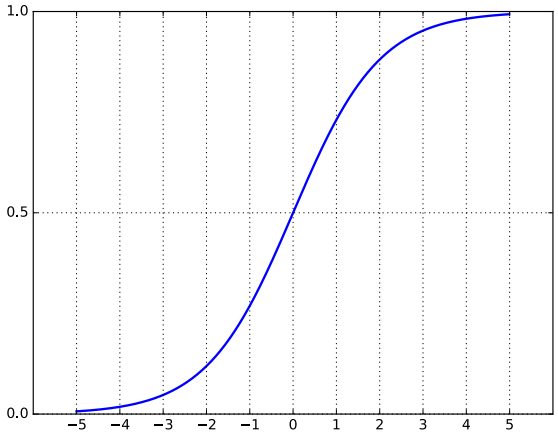
What needs to be chosen

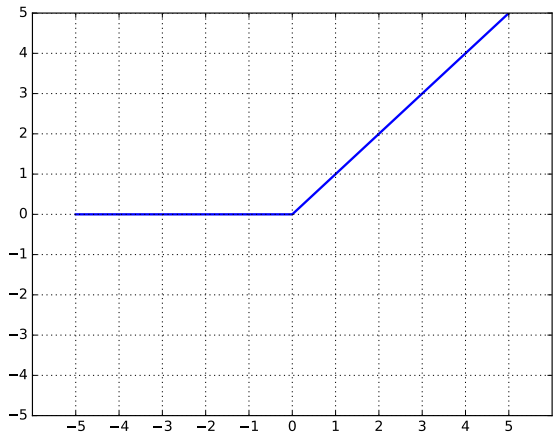
- Activation $\sigma$
- Number of layers $L$
- Number of neurons per hidden layer $n_i, i = 1, \ldots, L$.

Vocabulary

- Neuron
- Activation
- Hidden Layer
- Width, Depth
- Fully connected layer

# 2 possible activations: Sigmoid and Rectified Linear Unit




## Sigmoid function

- $x \mapsto \frac{\exp(x)}{1+\exp(x)}$
- Problems:
  1. Saturated function: gradient killer -> need for rescaling data

not to be used

## Rectified Linear Unit (ReLU)

- $x \mapsto \max(0, x)$
- Pros & cons:
  1. Not a saturated function. Kills negative values.
  2. Empirically, convergence is faster than sigmoid/tanh.
  3. Plus: biologically plausible

# How to deal with multi-class output: softmax activation

When we do multi-class classification, i.e., $Y_i \in 1, \ldots, K$, we output $K$ different values:

- Each of them corresponds to the probability of belonging to the class $j$, $1 \leq j \leq K$
- To obtain probabilities (adding up to 1):
  1. We have $K$ neurons on the last layer
  2. And use a Softmax activation to renormalize.

Softmax output unit, used to predict $\{1, \ldots, K\}$:

$$\text{softmax}(z)_i = \frac{e^{z_i}}{\sum_{k=1}^{K} e^{z_k}}$$

K neurons

last layer $z_1$ $z_n$ → softmax → $p_1$ $p_2$ : $p_K$

one prob per digit

Summary: if you have K classes (K = K for digits)

change the last layer activate the loss

Up to that point, we have seen :

- What a Neural Network was
- Why it is expected to learn better.

Next Question: how to implement it ?

# Python - Keras

1. Hardware:
   - CPU
   - GPU
   - TPU

2. Software (Python packages):
   - Pytorch/Tensorflow
   - → Keras : TF high level API. Ideal for applications.


# Keras - A few lines ![1]

What do we need to specify?

* architecture: $L$, $n_1 \dots n_2$ (layers; neurons on layer $1 \dots L$)
* activation
* regularizer
* properties of learning algo

What do we need to do next?

```python
# import tensorflow and keras (tf.keras not "vanilla" Keras)
import tensorflow as tf
from tensorflow import keras
# get data
(train_images, train_labels), (test_images,
                               test_labels) = keras.datasets.mnist.load_data()
# setup model
model = keras.Sequential([
    keras.layers.Flatten(input_shape=(28, 28)),
    keras.layers.Dense(128, activation=tf.nn.relu),
    keras.layers.Dense(64, activation= tf.nn.relu),
    keras.layers.Dense(10, activation=tf.nn.softmax)
])

model.compile(optimizer=tf.optimizers.Adam(),
              loss='sparse_categorical_crossentropy',
              metrics=['accuracy'])
# train models
model.fit(train_images, train_labels, epochs=5)

print('\n Evaluation' )
# evaluate
test_loss, test_acc = model.evaluate(test_images, test_labels)
print('test accuracy:', test_acc)
# make predictions
predictions = model.predict(test_images)
```

Wooclap!

- how many layers ?
- what is the final test accuracy ?
- what happens if you remove the hidden layers?
- how many parameters and why?
- how many points in the train set?
- what does 1865 and 313 correspond to?

# Wooclap

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What can you say about this code bit

1. The flatten layer is a dense layer
2. I have 2 hidden layers
3. I have one hidden layer
4. I should use sigmoid activation on the last layer
5. I use SGD for the algorithm
6. AdamOptimizer is EveOptimizer's boyfriend
7. I run 5 epochs of the stochastic algorithm
8. I use relu activation on all but the last layer

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```

```
Epoch 1/5
1875/1875 ━━━━━━━━━━━━━━━━━━━━ 6s 3ms/step - accuracy: 0.8124 - loss: 4.6972
Epoch 2/5
1875/1875 ━━━━━━━━━━━━━━━━━━━━ 4s 2ms/step - accuracy: 0.9239 - loss: 0.3186
Epoch 3/5
1875/1875 ━━━━━━━━━━━━━━━━━━━━ 5s 2ms/step - accuracy: 0.9464 - loss: 0.2017
Epoch 4/5
1875/1875 ━━━━━━━━━━━━━━━━━━━━ 5s 2ms/step - accuracy: 0.9548 - loss: 0.1581
Epoch 5/5
1875/1875 ━━━━━━━━━━━━━━━━━━━━ 4s 2ms/step - accuracy: 0.9610 - loss: 0.1388

 Evaluation
313/313 ━━━━━━━━━━━━━━━━━━━━ 1s 3ms/step - accuracy: 0.9592 - loss: 0.1602
test accuracy: 0.9621999859809875
313/313 ━━━━━━━━━━━━━━━━━━━━ 1s 2ms/step
```

at each step I look at a fraction (a batch) of data by default, 32 (in tf.fit)

5 epochs | 1875 steps x 32 batchsize ) 60k images in

We now see the output. What can we say

1. We overfit
2. There are 313 points in the test set
3. There are 10 000 points in the test set
4. 1875 is about 6 times 313
5. the loss is the accuracy
6. the loss decreases along the trajectory

# Wooclap

```
model.summary()
```

```
Model: "sequential"
_________________________________________________________________
 Layer (type)                Output Shape              Param #
=================================================================
 flatten (Flatten)           (None, 784)               0

 dense (Dense)               (None, 128)               100480

 dense_1 (Dense)             (None, 64)                8256

 dense_2 (Dense)             (None, 10)                650

=================================================================
Total params: 109,386
Trainable params: 109,386
Non-trainable params: 0
_________________________________________________________________
```

Optimizer parameters

Adam → total par

218 . 774

+2

2×

3 × 109 . 386

+2

SGD : 2!

Explain the number of parameters on each layer


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Total params: 109,386
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_________________________________________________________________
```


785 × 128 = 100 480

784 × 128 + 128 biases

129 × 64 = 8256

65 × 10 = 650

128 × 64 + 1 × 64

129 × 64

one "vector for momentum" of size 109k

one "vector of size for step size 109k"

# Outline

# Intuition

Up to that point, we have seen :

- What a Neural Network was.
- How to implement it in Python.

Next Question: why and when does it work?

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1. Approximation theorem
2. Optimization

# Approximation Theorem

Goal

$$\min_{f \in \mathscr{F}_{\text{MLP}}} \frac{1}{n} \sum_{i=1}^{n} [\ell(f(X_i), Y_i)].$$

As said before, a MLP can output non-linear functions... But how powerful is it?

# Approximation Theorem

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As said before, a MLP can output non-linear functions... But how powerful is it?

Continuous Neural Networks with one single hidden layer and any bounded and non constant activation function can approximate any function in $L^p$, provided a sufficient number of hidden units.

$\looparrowright$ Very powerful in terms of approximations.
Not very surprising: non linear function with millions of parameters!

# Optimization - Fitting the network

Goal $$\min_{f \in \mathscr{F}_{\text{MLP}}} \frac{1}{n} \sum_{i=1}^{n} [\ell(f(X_i), Y_i)].$$

How to minimize a function?
Gradient Methods. = cheapest way to update (learn) the weights iteratively to minimize the loss.

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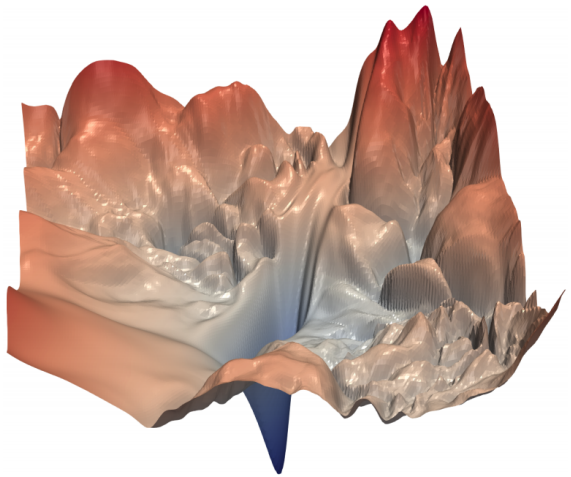
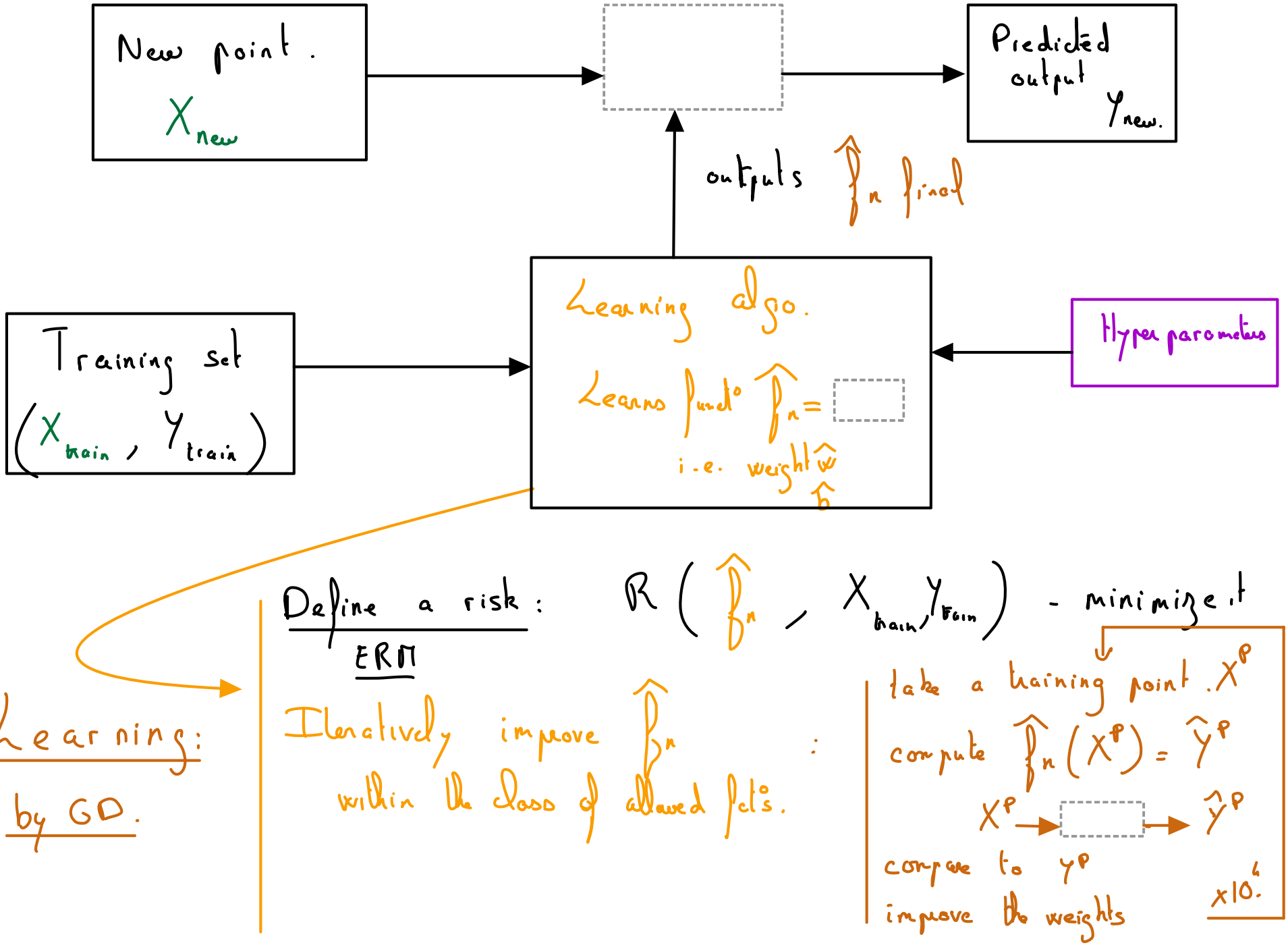
Two "miracles":

1. Autodifferentiation: even though the function is very complex and high dimensional, it is possible to compute gradients!
2. Optimization seems to work ok, though the function is non convex - we do not end up is too bad local minima. (specialized optim algos: Adam, AdaDelta, etc.)



These 2 miracles make it possible to learn a good regressor/classifier with NN and to benefit from the powerful approximation properties


New point.
$X_{new}$
Predicted output
$Y_{new}$
outputs $\hat{f}_n$ final
Learning algo.
Learns funct° $\hat{f}_n$ =
i.e. weight $\hat{w}$
$\hat{b}$
Training set
$(X_{train}, Y_{train})$
Hyperparameters
Learning:
by GD.
Define a risk:
ERM
$R(\hat{f}_n, X_{train}, Y_{train})$ - minimize it
Iteratively improve $\hat{f}_n$ within the class of allowed fct°.
take a training point $X^p$
compute $\hat{f}_n(X^p) = \hat{Y}^p$
$X^p$ → → $\hat{Y}^p$
compare to $Y^p$
improve the weights
x10.

# Conclusion

Neural networks :

1. learn very complex non linear functions in high dimension
2. can approximate nearly any function
3. can be optimized even though highly non convex.

Are thus expected to work:

- When depth or width increases.
- Thus with very large datasets
- Requiring a lot of computational power.

⇒ **Explains why they were so successful since 2010**

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*What my layer wants me to say*: we haven't talked about many points !!

- Initialization
- Regularization

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Next:

- Brief history
- How to use the structure?

# A brief history of NN

- 1943: Neural networks
- 1957: Perceptron
- 1974-86: Backpropagation, RBM, RNN
- 1989-98: CNN, MNIST
- 2009: ImageNet
- 2012: AlexNet,
- 2014: GANss
- 2016: AlphaGo
- 2018: BERT

The Computational power made the major change (+ investment and creativity).

# Directions

- When does it work?

**Large or huge datasets. Structured** tasks.

↬ **Images and text.**

⚠do not try to apply DL everywhere

Each application requires a special architecture:

- Images : Convolutional Neural Network (CNN).
- Text : Recurrent Neural Networks (RNN) (2012-2018), Transformers (2018-2024).

2017